Now try calculating the same slab, with the minimum recommended balanced load of 50% of self-weight. (As opposed to 50% of DL + LL used in the first calculation). The same thickness of slab will be used, with the same layout.

Given:— 4 bays each way 8000 x 7000
- 1.0 kPa finish
- 1.8 kPa partitions
- 2.5 kPa live load
- Slab will be protected from weather
- Internal columns 500 x 500
- External columns 300 x 500
- Wind loads taken by shear walls

**Ult. Load factors**
- DL 1.2 and 1.5 as SABS 0160-1989
- LL 1.6

**Floor to floor 3.0m**

18 mm sheath stress to 80% Ul

**CALCULATION**

Assume a 210 thick slab.

Balance 50% of Permanent Load

8.05 kPa/2 for internal spans

For external 8 m spans, balance same proportion of load as previously, i.e. 80/66 x 0.5 = 0.606 Permanent load

From the formula, the slab should be somewhat thicker than 230mm, but take 210 as minimum practicable.

\[
\text{self DL} = 5.25 \text{ kPa} \\
\text{Total perm} = 8.05 \\
\text{Service (1.1)} = 8.85 \\
\text{Live load} = 2.50 \\
\text{TOTAL} = 11.35
\]

Check for shear as before, 210 is a bit thin, but acceptable with capitals

As contractor has column shutters with capitals, use capitals:

For analysis assume E of slab = code value: i.e. (20 + 0.2 x 30) = 26.0 GPa

From the cross-sections, take the effective top cover as 47 mm to C/L cable for 7 m span, and (63 + 31)/2 = 47 mm for 8 m span. As bottom cover is 41 mm for 8 m span, effective drape = 210 - 88 mm = 122 mm.

For 7 m span, drape is 210 - 98 = 112 mm. (See Diag. Calculation 1)

From the diagrams of the cable shapes in the end spans (the cables in centre spans will have the same positions as at the ends of outer spans), the properties of the parabolas may be deduced, and then the losses due to curvature.

Referring to the formula in Appendix A, the following table may be calculated, as before.

<table>
<thead>
<tr>
<th>Span</th>
<th>( a_1 )</th>
<th>( a_2 )</th>
<th>( b_1 )</th>
<th>( b )</th>
<th>( b_3 )</th>
<th>( L )</th>
<th>( \lambda )</th>
<th>( m )</th>
<th>( n )</th>
<th>( X )</th>
<th>( c_1 )</th>
<th>( c_2 )</th>
<th>Drape</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.0 end</td>
<td>0.4</td>
<td>0.4</td>
<td>0.1</td>
<td>0.041</td>
<td>0.163</td>
<td>8.0</td>
<td>0.0330</td>
<td>1.34</td>
<td>-5.411</td>
<td>3.7018</td>
<td>0.0096</td>
<td>0.0114</td>
<td>0.0944</td>
</tr>
<tr>
<td>8.0 cent</td>
<td>0.4</td>
<td>0.4</td>
<td>0.163</td>
<td>0.041</td>
<td>0.163</td>
<td>8.0</td>
<td>0.0</td>
<td>1.854</td>
<td>-7.418</td>
<td>4.0</td>
<td>0.0122</td>
<td>0.0122</td>
<td>1.098</td>
</tr>
<tr>
<td>7.0 end</td>
<td>0.35</td>
<td>0.35</td>
<td>0.13</td>
<td>0.051</td>
<td>0.163</td>
<td>7.0</td>
<td>0.0330</td>
<td>1.039</td>
<td>-3.677</td>
<td>3.2114</td>
<td>0.0096</td>
<td>0.0033</td>
<td>0.0853</td>
</tr>
<tr>
<td>7.0 cent</td>
<td>0.35</td>
<td>0.35</td>
<td>0.163</td>
<td>0.051</td>
<td>0.163</td>
<td>7.0</td>
<td>0.0</td>
<td>149</td>
<td>-5.214</td>
<td>3.5</td>
<td>0.0112</td>
<td>0.0112</td>
<td>1.008</td>
</tr>
</tbody>
</table>

**CALCULATION**

Loads to be Balanced:

For 8 m span, balance 0.606 x permanent load in external span:

For normal external bay 7 x 8.05 x 0.606 = 34.15 kN/m

For 8 m internal span, balance 0.5 x permanent load
i.e. 7 x 8.05 x 5 = 28.17 kN/m

For 7 m span, balance 0.50 x permanent load in external & internal span.

For internal and external bay, 8 x 8.05 x .5 = 32.20 kN

**COMMENT**

See page 1

For first internal bay, increase cables to allow for increased load at first internal support
The loads to be balanced, the required prestress force = \( \frac{wL^2}{8xDrape} \). This is the final force after losses, and assuming a loss of say 18% for 8m spans, and 16% for 7m spans, the initial prestress can be calculated. The angle which the cable rotates through is also required for the friction calculation. They are calculated from the formula for a parabola.

\[ e.g. \text{for the 8m end span} \theta = \arctan \left( \frac{2 \times 78.4}{3268.4} \right) \times 2 + \arctan \left( \frac{2 \times 113.5}{3931.6} \right) \times 2 = 0.2113 \text{ Radians} \]

The following table shows the calculation results.

<table>
<thead>
<tr>
<th>Span</th>
<th>Load</th>
<th>Drape</th>
<th>Eff. Length</th>
<th>Angle to C/L</th>
<th>Angle C/L to Total angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 m end span</td>
<td>34.15</td>
<td>0.0944</td>
<td>7.2</td>
<td>0.09615</td>
<td>0.1135</td>
</tr>
<tr>
<td>8m 2nd span</td>
<td>28.17</td>
<td>0.1098</td>
<td>7.2</td>
<td>0.122</td>
<td>0.122</td>
</tr>
<tr>
<td>8m 3rd span</td>
<td>do</td>
<td>do</td>
<td>do</td>
<td>do</td>
<td>do</td>
</tr>
<tr>
<td>8m last span</td>
<td>as 1st</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7m 1st span</td>
<td>32.2</td>
<td>0.0853</td>
<td>6.3</td>
<td>0.09838</td>
<td>0.1182</td>
</tr>
<tr>
<td>7m 2nd span</td>
<td>32.2</td>
<td>0.1008</td>
<td>6.3</td>
<td>0.128</td>
<td>0.128</td>
</tr>
<tr>
<td>7m 3rd span</td>
<td>do</td>
<td>do</td>
<td>do</td>
<td>do</td>
<td>do</td>
</tr>
<tr>
<td>7m 4th span</td>
<td>as 1st</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Loses:
8 m spans
The total loss due to friction and wobble is
\[ \text{factor} = e^{-0.025 \times 32.0 \times 0.06 \times 0.9073} = 0.8742 \]

i.e. 12.58% losses.

7m spans
The total loss due to friction and wobble is
\[ \text{factor} = e^{-0.025 \times 28.0 \times 0.06 \times 0.9452} = 0.8810 \]

i.e. 11.9% losses.

**CALCULATION**

8 m spans: end span
The prestress required to balance 59 kN/m with a drape of 0.0951, and an effective length of parabola of 8 - 0.8 x 7.2 m is given by
\[ 34.15 \times \frac{7.2^2}{8 \times 0.0944} = 2344 \text{ kN} \]

allowing an estimated 18% losses, \( P_{\text{init}} = 2859 \text{ kN} \)
If cables of 15.2mm dia (ult. strength 260 kN) are stressed to 80% of ult. the no. of cables required
\[ \frac{2859}{0.8 \times 260} = 13.7 \text{ say 14 cables} \]

8m spans: 2nd span
The prestress required to balance 28.17 kN with drape of .1122 =
\[ \frac{28.17 \times 7.2^2}{8 \times 0.1122} = 1627 \text{ kN} \]

Allowing the same losses, \( P_{\text{init}} = 1984 \text{ kN} \), and no of cables = 
\[ \frac{1984}{208} = 9.5 \text{ cables} \]

say 10.
Assume that the cables in the end span are taken to 1.5 m past the 1st support, and that half the cables are stressed from each end, the diagrams on the next page may be drawn. Although strictly one should calculate the losses from the exponential equation it is quite acceptable to use a linear method.
An even number is better if stressed from both ends.
8 m Spans  Effects of friction
Stressing forces in cables before other losses

Diagram showing force in one cable, after friction, and before pull-in

Diagram showing force in one cable, after pull-in

Total 9 Cables

Force in 9 and 5 cables stressed from one end

Force in 14 and 10 cables (half stressed from each end)
7 m Spans  Effects of friction
Stressing forces in cables before other losses

Diagram showing force in one cable after friction and before pull-in

Diagram showing force in one cable after pull-in

Total 7 Cables
Total 5 Cables

Force in 7 and 5 cables stressed from one end

Total Force in 12 and 10 cables (half cables stressed from each end)
For 7m external span, force to balance 32.20 kN/m, with drape of 0.0853, and effective length of 6.3m = 1873 kN.

With 16% losses, no of cables = 10.8 say 12.

For internal span, force to balance 34.15 kN/m with drape of 0.1008, \( P_{\text{final}} = 1585 \text{ kN} \rightarrow 1887 \text{ kN} \) before losses.

No. of cables = 9.0 say 10.

Take losses due to friction and pull-in as before (see first calculation).

The results are diagrammed on pages 5 and 6.

### Other Losses:

#### Elastic Shortening

8m spans

Average stress in end 9.5 m = \((2590 + 2699)/(7 \times 0.21 \times 2)\) = 1.80 MPa

In Centre part \((1927 + 1949)/2 \times 3.52 + (1949 \times 2.98)/6.5(7 \times 0.21)\) = 1.32 MPa

Losses in end span = 1.80 x 198/(26 x 2) = 6.85 MPa

Average loss over whole span = \((1.80 \times 9.5 \times 2 + 1.32 \times 13)/(2 \times 32)\) x 198726 = 6.11 MPa

Loss per cable = 140 x \(10^{-6}\) x 6.85 x 1000 kN = 0.96 kN in end spans (x 4 = 3.8 kN)

Total in end spans = 12.4 kN

7m spans

Average stress whole length = \((2223 + 2313.6) \times 8.5 + (1925 + 1956)/2 \times 2.87 + 1956 \times 1.63)/(14 \times 8 \times 0.21)\) = 1.19 MPa

In end average force = \(=(2223 + 2313.6)/2 = 2268.2 \rightarrow 1.35 \text{ MPa}\)

Losses 10 cables per cable = 1.19 x 198/(26 x 2) x 140 x \(10^{-6}\) x 1000 = .63 kN

Loss in 2 cables = 1.35 x 140 x \(10^{-6}\) x 198/26 = 0.72 kN

10 x .63 + 2 x .72 = 7.8 kN total

Total in centre span = 6.3 kN

The resultant forces after friction and elastic losses are shown in the diagram below.
**CALCULATION**

**Long term losses**

Relaxation 1.5% x 2 = 3%

For 8m spans:

- end span 3% x \((2578+2687)/2\) = 79 kN
- centre 3% x \((1918+1940)/2\) = 91 kN

For 7m spans:

- end span 3% x 2,252 = 67.6 kN
- centre 3% x 1,950 = 58.5 kN

Creep. (Same creep factor as 1st calc)

If stressed at 3 days (common for prestressed flat slabs) and with a humidity of 45%, the creep for a 210 slab, interpolating between 150 and 300 thick slabs given in the code (BS8110) gives a creep factor of 3.7

Then loss = stress in concrete x ratio of moduli x 3.7

8m spans:

- End Section conc. stress = \((2.578 + 2.687)/(2*7\times 0.21)\) = 1.79 MPa
- Loss of steel stress = 1.79 x 3.7 x 198/26 = 30 MPa
- Per cable 50.5 x \(x\) Area = 7.06 kN
- Centre Section \((1.918+1.94)/3.52 + (1.94 x 2.98)/6.5\) x 198/26 x 3.7 = 37.1 MPa
- Per cable 37.1 x 0.14 = 5.19 kN Therefore average loss in long cables

Total creep loss:

- End 7.06 x 4 + 6.3 x 10 = 91.2 kN
- Centre 6.3 x 10 = 63 kN

7m spans:

- End Section conc. stress after elastic losses\((2215.1 + 2305.8)/(2\times 8\times 0.21)\) = 1.345 MPa
- Centre \((1918.7 + 1947.7)/(2\times 8\times 1.63)/(4.5 \times 8\times 2.1)\) = 1.154 MPa
- Average\((1.154 \times 5.5 + 1.345 \times 8.5)/14\) = 1.27 MPa
- Loss of steel stress : end 1.345 x 3.7 x 198/26 = 37.9 MPa
- Per cable 37.9 x 0.14 = 5.306 kN
- Centre loss of steel stress 1.27 x 3.7 x 198/26 = 35.78 MPa
- loss/cable = 35.7 x 0.14 = 5.0 kN
- Losses in centre 10 x 5.0 = 50 kN
- Losses in end section 12 x 5.306 = 63.7 kN

Shrinkage as before: for 1 cable 10.26 kN

8m spans

- 14 cables = 143.6 kN
- 10 cables = 102.6 kN

7m spans

- 12 cables = 123.1 kN
- 10 cables = 102.6 kN

<table>
<thead>
<tr>
<th>Cause</th>
<th>8m End span</th>
<th>8m Centre</th>
<th>7m End span</th>
<th>7m Centre</th>
</tr>
</thead>
<tbody>
<tr>
<td>No of cables</td>
<td>14</td>
<td>10</td>
<td>12</td>
<td>10</td>
</tr>
<tr>
<td>Initial force (208kN/cable)</td>
<td>2912 kN</td>
<td>2080</td>
<td>2496</td>
<td>2080</td>
</tr>
<tr>
<td>Elastic</td>
<td>12.4 kN</td>
<td>8.6</td>
<td>7.8</td>
<td>6.3</td>
</tr>
<tr>
<td>Relaxation</td>
<td>79 kN</td>
<td>58</td>
<td>67.6</td>
<td>58.5</td>
</tr>
<tr>
<td>Shrinkage</td>
<td>143.6</td>
<td>102.6</td>
<td>123.1</td>
<td>102.6</td>
</tr>
<tr>
<td>Creep</td>
<td>91.2</td>
<td>63</td>
<td>63.7</td>
<td>50</td>
</tr>
<tr>
<td>TOTAL</td>
<td>321.2</td>
<td>232</td>
<td>262</td>
<td>217.4</td>
</tr>
<tr>
<td>Percentage of init. force loss after friction loss</td>
<td>11.1</td>
<td>11.1</td>
<td>10.5</td>
<td>10.4</td>
</tr>
<tr>
<td>% Loss due to friction</td>
<td>11.4</td>
<td>7.45</td>
<td>11.08</td>
<td>8.31</td>
</tr>
<tr>
<td>Total % loss of initial</td>
<td>22.5</td>
<td>18.5</td>
<td>21.6</td>
<td>18.7</td>
</tr>
</tbody>
</table>

The losses are somewhat lower than with the higher prestress but still appreciably higher than the 16% (or less) assumed by some commercial designers.

From the figures for long-term loss, the final prestress can be calculated.
The slabs are analysed by the equivalent frame method, using Long's method to calculate the equivalent stiffness of the columns. Three loading cases are needed: Dead load on all spans, Line load on even spans, and live load on odd spans. These may be combined together with the load factors to give the desired bending moments diagrams. Any method of analysis may be used: moment distribution if a computer is not available, or a frame analysis if one is available. The hogging moments for the ultimate limit state may then be reduced by 15%, and the sagging moments increased accordingly to maintain equilibrium.

The design moment is at the face of the column or capital, but the total statically required moment is: \( W(L-2D/3)/8 \) If a computer program is used, there is an advantage in arranging a node at 1/3 of the column or capital dimension from the centreline of the column, as the moment at that point will not be less than the moment given for statically required moment.

Loads (As for previous calculation)

<table>
<thead>
<tr>
<th>LOAD</th>
<th>Service (1.1DL + LL)</th>
<th>Ultimate (1.2DL + 1.6LL)</th>
<th>Ultimate Dead Load (1.5 DL)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Finishes</td>
<td>10 kPa</td>
<td>12.0kPa</td>
<td>1.5kPa</td>
</tr>
<tr>
<td>Partitions</td>
<td>1.98</td>
<td>2.16</td>
<td>2.7 kPa</td>
</tr>
<tr>
<td>Self. Wt</td>
<td>5.77</td>
<td>6.30</td>
<td>7.87kPa</td>
</tr>
<tr>
<td>Total DL</td>
<td>8.85</td>
<td>9.66</td>
<td>12.07kPa</td>
</tr>
<tr>
<td>Live load</td>
<td>2.50</td>
<td>4.00</td>
<td></td>
</tr>
</tbody>
</table>

Total loads on spans

<table>
<thead>
<tr>
<th></th>
<th>Service</th>
<th>Ultimate</th>
<th>Self only</th>
</tr>
</thead>
<tbody>
<tr>
<td>8m Spans DL</td>
<td>61.95kn/m</td>
<td>67.62kN/m</td>
<td>36.75</td>
</tr>
<tr>
<td>8m spans LL</td>
<td>17.5</td>
<td>28.0</td>
<td>36.75</td>
</tr>
<tr>
<td>7m spans DL</td>
<td>70.8</td>
<td>77.28</td>
<td>42.0</td>
</tr>
<tr>
<td>7m spans LL</td>
<td>20.0</td>
<td>32.0</td>
<td>42.0</td>
</tr>
</tbody>
</table>

NOTE: As the Ultimate Deadload case of 1.5DL at 12.07 kPa is less than the Ultimate DL + LL of 13.66 kPa, it may be effectively ignored, although there may be some places where the moments could be fractionally higher. A construction loading at initial prestress may need to be calculated if the propping is not adequately arranged.

Loads due to prestress
For a parabola, the equivalent uniform load caused by a tension in the cable is given by \( wL^2h/8 = P h \) where \( h \) is the drape for the cables. The drapes can be read from the table on page 1. The loads will not be uniform, but will be trapezoidal if the variation of prestress along the beam is taken into account. In addition the moments due to eccentricity at the ends must be taken into account in the analysis if the cables are not exactly central. In our case we have assumed a 20mm eccentricity upwards at the end:

The moment at the end of the 8m spans will be 2267 x .02 = 56.7 kNm

The moment at the end of the 7m spans will be 1959 x .02 = 49.0kNm

The loads given are for final prestress. Initial prestress (after friction and elastic losses) combined with deadload only may be a critical state, but it is probably sufficient to take the stresses due to final prestress and multiply them by an average factor.
Note: The forces at intermediate points are interpolated.

### Table: Load Distribution

<table>
<thead>
<tr>
<th>Position</th>
<th>Prestress</th>
<th>Drape</th>
<th>Length</th>
<th>Lateral load</th>
</tr>
</thead>
<tbody>
<tr>
<td>8m 1st span L end</td>
<td>2267</td>
<td>.0096</td>
<td>0.8</td>
<td>272.0 kN/m down</td>
</tr>
<tr>
<td>8m 1st start of sag</td>
<td>2272</td>
<td>.0944</td>
<td>7.2</td>
<td>33.1 up</td>
</tr>
<tr>
<td>8m 1st, end of sag</td>
<td>2353</td>
<td>.0944</td>
<td>7.2</td>
<td>34.28 do-</td>
</tr>
<tr>
<td>8m 1st span RH end</td>
<td>2358</td>
<td>.0114</td>
<td>0.8</td>
<td>336.0 kN/m down</td>
</tr>
<tr>
<td>8m 2nd span L end</td>
<td>2354</td>
<td>.0122</td>
<td>0.8</td>
<td>399.6 down</td>
</tr>
<tr>
<td>-do- end of cable</td>
<td>2376</td>
<td>.1098</td>
<td>7.2</td>
<td>40.26 up</td>
</tr>
<tr>
<td>-do- beyond end</td>
<td>1717</td>
<td>.1098</td>
<td>7.2</td>
<td>29.1 up</td>
</tr>
<tr>
<td>8m 2nd RH end</td>
<td>1717</td>
<td>.0122</td>
<td>0.8</td>
<td>261.8 kN/m down</td>
</tr>
<tr>
<td>7m 1st span LH</td>
<td>1959</td>
<td>.0086</td>
<td>0.7</td>
<td>275.1 down</td>
</tr>
<tr>
<td>7m 1st: start of sag</td>
<td>1966</td>
<td>.0853</td>
<td>6.3</td>
<td>33.80 up</td>
</tr>
<tr>
<td>7m 1st end of sag</td>
<td>2026</td>
<td>.0853</td>
<td>6.3</td>
<td>34.83 up</td>
</tr>
<tr>
<td>7m 1st RH</td>
<td>2034</td>
<td>.0103</td>
<td>0.7</td>
<td>342.0 down</td>
</tr>
<tr>
<td>7m 2nd LH end</td>
<td>2034</td>
<td>.0122</td>
<td>0.7</td>
<td>405.1 down</td>
</tr>
<tr>
<td>7m 2nd: start of cable</td>
<td>2050</td>
<td>.1008</td>
<td>6.3</td>
<td>41.65 up</td>
</tr>
<tr>
<td>7m 2nd end of cable</td>
<td>1709</td>
<td>.1008</td>
<td>6.3</td>
<td>34.72 up</td>
</tr>
<tr>
<td>7m 2nd Rh end</td>
<td>1738</td>
<td>.0122</td>
<td>0.7</td>
<td>346.2 kN/m down</td>
</tr>
</tbody>
</table>

### Diagram: Load Distribution

#### Loads on 8m spans due to final prestress
- **272 kN/m**
- **336 359.6 261.6**

#### Loads on 7m spans due to final prestress
- **275.1 kN/m**
- **342 405.1 346.2**

### Equations:

- **Columns**: Stiffness by Long's method (See Sample Calculation 1)
- **7m spans Exterior**: E (30 MPa) = 26 GPa
  - \( f = \frac{5 \times 3^2 \times 12}{2} = 1.125 \times 10^3 \) m units
  - \( K_e = 4EI/L_c \)
  - \( K_c = K_e/(1 + 1.1272K_eL/Eh_c) \)
  - \( e = K_e/(1 + 1.1272(4I_c/L_c)h^3_c) \)
where $L=8\text{m}$, $L_c=3$, $h=21\text{.}$ and $c=3$

Then equivalent stiffness = stiffness $\times$ 0.645

$8\text{m spans interior}$: $I_c = \frac{5.9}{12} = 5.208 \times 10^{-3}$, $k = 0.564$

c = 0.5

Then equivalent stiffness = stiffness $\times$ 0.645

$7\text{m exterior}$: $I_c = \frac{3 \times 5.5}{12} = 0.125 \times 10^{-3}$

c = 0.5

Then equivalent stiffness = stiffness $\times$ 0.596

Equiv $I = 3.10 \times 10^{-3}$

$7\text{m interior}$: $I_c = \frac{5.208 \times 10^{-3}}{3}$, $c=0.5$, $k=0.628$

Then equivalent stiffness = stiffness $\times$ 0.555

Equiv $I = 1.73 \times 10^{-3}$

Slab Stiffness

The moment of Inertia of the $8\text{m}$ spans is $7 \times 0.21^3/12 = 0.0054 \text{m}^4$

The moment of inertia of the $7\text{m}$ spans is $8 \times 0.21^3/12 = 0.00617 \text{m}^4$

The $E$ is taken as the same as the columns, and the creep factor (for calculation deflections due to long term loads) is 3.5

The results of the analysis are given below:

Because the structure is symmetrical, only the first two spans are shown.

It should be realised that as the loads are calculated for an interior span, the first interior column band should be reinforced for approximately a 7% greater load.

i.e. for this design, 1 extra prestressing cable, and some additional reinforcement over the columns.

<table>
<thead>
<tr>
<th>MOMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>8m 1st span Left</td>
</tr>
<tr>
<td>8m 1st span centre</td>
</tr>
<tr>
<td>8m 1st span right</td>
</tr>
<tr>
<td>8m 2nd span left</td>
</tr>
<tr>
<td>8m 2nd span centre</td>
</tr>
<tr>
<td>8m 2nd span right</td>
</tr>
<tr>
<td>7m 1st span left</td>
</tr>
<tr>
<td>7m 1st span centre</td>
</tr>
<tr>
<td>7m 1st span right</td>
</tr>
<tr>
<td>7m 2nd span left</td>
</tr>
<tr>
<td>7m 2nd span centre</td>
</tr>
<tr>
<td>7m 2nd span right</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DEFLECTIONS (allowing 3.5 creep factor on permanent loads)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>8m 1st span</td>
</tr>
<tr>
<td>8m 2nd span</td>
</tr>
<tr>
<td>7m 1st span</td>
</tr>
<tr>
<td>7m 2nd span</td>
</tr>
</tbody>
</table>

Serviceability Limit State

For controlling cracking, it has been traditional to limit tensile stresses. For the sake of completeness, this will be done, but the incremental stress method is considered better. (Use the program described in Appendix E)

The permissible tensile stress is given as $0.45\sqrt{f_c}$ for 2.46 MPa

For controlling cracking, it has been traditional to limit tensile stresses.
SAMPLE CALCULATION 2 Page 10

Serviceability Tensile stress

<table>
<thead>
<tr>
<th>Position</th>
<th>Net moment (service-prestress)</th>
<th>Comp. kN</th>
<th>Stress (moment)</th>
<th>Stress (comp)</th>
<th>Net tensile stress (MPa)</th>
<th>Z of section (M units)</th>
<th>Slab Area (M units)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8m 1st L</td>
<td>46.4</td>
<td>2267</td>
<td>±0.90</td>
<td>1.54</td>
<td></td>
<td>0.0514</td>
<td>1.47</td>
</tr>
<tr>
<td>8m 1st C</td>
<td>201.8</td>
<td>2358</td>
<td>3.93</td>
<td>1.57</td>
<td>4.04*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8m 1st R</td>
<td>-290.0</td>
<td>2376</td>
<td>5.64</td>
<td>1.60</td>
<td>-248.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8m 2nd L</td>
<td>-248.1</td>
<td>2376</td>
<td>4.83</td>
<td>1.60</td>
<td>-323*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8m 2nd C</td>
<td>116.8</td>
<td>1717</td>
<td>2.27</td>
<td>1.17</td>
<td>2.32</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8m 2nd R</td>
<td>179.4</td>
<td>1717</td>
<td>3.49</td>
<td>1.17</td>
<td>-179.4</td>
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<td></td>
</tr>
<tr>
<td>7m 1st L</td>
<td>17.7</td>
<td>1959</td>
<td>0.30</td>
<td>1.17</td>
<td>-244.8</td>
<td>-0.0588</td>
<td>1.68</td>
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<tr>
<td>7m 1st R</td>
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<td>2050</td>
<td>4.16</td>
<td>1.21</td>
<td>2.95*</td>
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<td></td>
</tr>
<tr>
<td>7m 2nd L</td>
<td>-214.5</td>
<td>2050</td>
<td>3.65</td>
<td>1.21</td>
<td>2.44</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7m 2nd C</td>
<td>96.9</td>
<td>1709</td>
<td>1.65</td>
<td>1.02</td>
<td>0.63</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7m 2nd R</td>
<td>-140.7</td>
<td>1738</td>
<td>2.39</td>
<td>1.03</td>
<td>1.36</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

It may be seen that 3 of the stresses are more than the Report 25 stresses, and reinforcement is required. The tension to be taken by reinforcement is calculated by simply taking the force from the tensile stress diagram as in the sketch at a stress of 0.58 \( f' \). It should be noted that it is assumed that the moments are evenly distributed across the section. This is clearly incorrect.

Ultimate Load Limit State.
15% reduction of hogging moments (with corresponding increase of sagging moments) is allowed and has been taken.

The decompression moment, equal to the prestress \( x Z \), is the moment required to reduce the compression on the extreme fibre to zero. If the applied moment (Ultimate dead + Live + prestress) is less than the decompression moment, no reinforce is required. If it is greater, an equivalent moment \( M' \) is calculated, by displacing the prestress to the level of the reinforcement, and adding a moment of \( P(d-h/2) \). The tension calculated from this moment then has the prestress force deducted, to obtain the net tension on the reinforcement \( M7Jd-P \). The required steel area is then \( (M'/Jd-P)/f' \).

The ultimate M.R. of concrete = 4651 bd^2 = 126 kNm for the 7m spans and 154 kNm for the 8m spans. None of the design moments

(Calculated using SABS0100 with 0.2mm crackwidth. See Appendix E) The areas are slightly greater than for the Report 25 Calculation (e.g. Y16 at 200 or Y12 @ 180 as compared with Y12 at 180)
Frame analysed

Ultimate moments for 8m spans from analysis (with 15% redistribution) (Note reduction to face)

Final Prestressing Moments for 8 m spans
exceeds this. The lever arm as a proportion of effective depth is given approximately by

\[ J = \left( 0.5 \cdot \sqrt{0.25 - \frac{0.171 M}{M_{db}}} \right) \]

\[ d - \frac{h}{2} = 65 \text{mm for the 8m spans, and 49mm for the 7m spans.} \]

The approximate moment per metre taken by Nominal reinforcement is \( 391 \text{ MPa} \times 0.75 \times 0.15 \times 0.21 \text{m} = 16.8 \text{ kNm for } d = 182 \text{ mm} \) and \( 15.3 \text{ kNm/m for } d = 166 \text{ mm} \).

The reinforcement areas required for ultimate load by this method are a bit smaller than those required for crack control. Compare the above calculation with the Report 25 method of calculating the reinforcement required for ultimate moments:

- This method does not take account of the moments due to prestress, but effectively assumes the cables are bonded, and that the moments are taken over the full width of the section.

At the first interior support of the 8m spans, \( f_{pc} = 2.3581 \text{MN/(14 x 140 x 10}^{-6} = 1203.1 \text{MPa (Effective prestress)} \)

\[ f_{pc} = f_{pu} + \frac{7000}{l} \left( 1 - \frac{1.7 f_{pc} A_{pc}}{f_{pc} b d} \right) \]

Effective depth of cables \( = 163 \text{mm} \) (see page 3 of calculations).

From equation 52 of BS8110 for \( l/d = 16/0.163 = 98 \), \( f_{pb} = 1203.1 + 61.2 = 1264 \text{ MPa} \)

For end 8m spans (SABS 0100) \( l=32/2=16 \)

For internal spans, \( l=32/3 = 10.67 \)

(For 8m int 1295MPa, for 7m end 1274, for 7m internal \( f_{pb} = 1302 \text{MPa} \))

<table>
<thead>
<tr>
<th>Position</th>
<th>Net. Moment (Ult.-prestress)</th>
<th>Net Moment /m in band (A)</th>
<th>Mom. / m (d-h/2) (B)</th>
<th>DecomMom/m</th>
<th>Prestress ( x Z )</th>
<th>Design Moment / m (A+B)</th>
<th>Net Tension kn/m</th>
<th>Bar Dia &amp; spacing</th>
<th>Area (sq.mm/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8m 1st L</td>
<td>49.3</td>
<td>10.6</td>
<td>21.0</td>
<td>11.3</td>
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<td>116.6</td>
<td>Y10@250</td>
<td>Nom</td>
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<tr>
<td>8m 1st C</td>
<td>80.3</td>
<td>47.7</td>
<td>21.4</td>
<td>69.1</td>
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<td>155.4</td>
<td>Y10@250</td>
<td>298</td>
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<tr>
<td>8m 1st R</td>
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<td>-58.7</td>
<td>21.8</td>
<td>80.5</td>
<td></td>
<td></td>
<td>155.4</td>
<td>Y10@200</td>
<td>398</td>
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<tr>
<td>8m 2nd L</td>
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<td>120.0</td>
<td>16.0</td>
<td>49.0</td>
<td></td>
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<td>121</td>
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</tr>
<tr>
<td>8m 2nd C</td>
<td>35.3</td>
<td>33.0</td>
<td>21.9</td>
<td>50.1</td>
<td></td>
<td></td>
<td>121</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7m 1st L</td>
<td>-23.2</td>
<td>4.3</td>
<td>12.0</td>
<td>47.5</td>
<td></td>
<td></td>
<td></td>
<td>Nom</td>
<td></td>
</tr>
<tr>
<td>7m 1st C</td>
<td>256.8</td>
<td>35.3</td>
<td>12.2</td>
<td>121</td>
<td></td>
<td></td>
<td></td>
<td>Nom</td>
<td></td>
</tr>
<tr>
<td>7m 1st R</td>
<td>-226.9</td>
<td>-42.5</td>
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<td>24.2</td>
<td>10.5</td>
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<tr>
<td>7m 2nd C</td>
<td>176.2</td>
<td>24.2</td>
<td>10.5</td>
<td>34.7</td>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>7m 2nd R</td>
<td>-119.5</td>
<td>-22.4</td>
<td>10.6</td>
<td>33.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The other positions are checked in the table below.
The reinforcement required for ultimate moments by this method are slightly less than required for cracking, and slightly less than the reinforcement required by the suggested method.

Ultimate MR of section = 537x .166 + 2473 x .147= 453 kNm

It is also desirable to check the ultimate load case with half of the cables in the end span removed, with DL + .25 LL, in the end spans only. The MR due to half the number of cables may be taken as half the M.R. in the table above if Report 25 method is used, or the moments calculated for prestress may be halved and deducted from the moments for DL + .25 LL.

The second method will be used

It may be seen that the reinforcement for the 1.2DL + 1.6LL ultimate load case is adequate everywhere except the first interior support of the 8m spans.

Deflections:
The deflections in the table above were calculated for a non-cracked slab. If the sections are cracked, the Moment of Inertia would be $2.56 \times 10^{-3}$ instead of the uncracked value of $5.4 \times 10^{-3}$.

Using the ACI formula, and taking moment at first cracking as the moment at which the tensile stress reaches 2MPa, the

\[
l_e = \left( \frac{M_{cr}}{M_D} \right)_l + \left[ 1 - \left( \frac{M_{cr}}{M_D} \right)_r \right]_e
\]

cracking moment in the 8m 1st span is $(1.54+2) \times 7 \times 21.21^2/6 = 182$ kNm. Taking the design moments as the serviceability moments, and taking the moments due to prestress into account, the moment at midspan of the 8m 1st span is 201.8 kNm. i.e. cracked, and at the first internal support 290 kNm.

Then $l_e$ midspan = $4.13 \times 10^{-3}$

$1 \times $ Support = $3.26 \times 10^{-1}$ using the service load moments calculated above.

The actual moment over the support is more concentrated, and there may be more cracking. The net equivalent M. of I. is then $85 \times 4.13 + .15 \times 3.26 = 40$

The calculated deflection will then be $5.4/4.0 \times$ the calculated one.

Now the calculated deflection assumes that the slab acts as a band spanning in one direction, and neglects the span in the other direction. If one adds half the calculated deflection in the short direction to the calculated deflection in the long direction, this will probably be a reasonable estimate.
The total deflection is then \((18.0 + 5.15) \times \frac{5.4}{4.0} = 31\text{mm}\) 

This is \(\frac{1}{223}\) of the short span, and may be acceptable if there are no rigid partitions. It should be noted that if there are reasons the slab might be more cracked, e.g. temperature stresses or shrinkage, the deflections could be considerably greater.

**Shear**: The calculated ultimate shear at the first interior support (at the face of the supports) is (from the computer calculations) \(841.95\text{ kN}\) (as compared to the simply supported load of \(765.9\text{ kN}\)). The shear assumed in the preliminary calculations was \(860\text{ kN}\)

The design shear is 1.15 times this, or \(968.9\text{ kN}\).

Alternatively, from the code, \(V_{\text{eff}} = V_t(1 + 1.5M/V_t x)\)

\(M_t\), the moment transmitted to the column, is \(226.0\text{ kNm}\), and \(x\) is \(1.5 + .18 \times 1.5\) (The column capital + 1.5 slab depths)

Then \(V_{\text{eff}} = 1.227 \times 841.95 = 1033.4\text{ kN}\)

From this may be deducted the vertical component of the **prestressing** cables.

At \(1.06\) m from the c/l of column, the slope of the cables is \(.0529\) from the properties of the parabola in the \(8\text{m span}\)

Vertical component = \(4/14 \times 2267 \times .0529 = 34\text{ kN} \times 2 = 68\text{ kN}\)

In the \(7\text{m span}\) the slope is \(.0575\)

Component = \(5/12 \times .0525 \times 2056 \text{ kN} = 44.9\text{ kN} \times 2 = 89.8\text{ kN}\)

Total vert. component on 4 sides = \(157.8\text{ kN}\) and net design shear = \(1033.4 - 157.8 = 875\text{ kN}\)

Area of reinforcement (\(Y_{10@200}\)) in one direction, and \(Y_{12} @ 180\) in the other. Average is \(510\text{ sq mm/m}\)

Average \% = \(100(1860 \times 4.5 \times 140 \times 10^{-6} + 510 \times 1.71 \times 10^{-6} \times 450)/(1.71 \times .21 \times 450) = 0.97\%\)

Permissible shear stress = \(.0.70\text{ MPa}\)

Actual shear stress = \(851/(4 \times 2.04 \times .167) = .63\text{ MPa}\). Therefore no shear reinf. reqd

At other columns, shear is less, and moment transferred to column is smaller. No shear reinforcement required.

One should also calculate that the width of slab at the external columns is adequate to transfer the moment, but with Long's method, the moment transferred is quite small, and with the column capitals, it is not necessary.