Report No. 2

Design of prestressed Concrete flat slabs

© Joint Structural Division of the South African Institution of Civil Engineering and the Institution of Structural Engineers

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Foreword

This Report is intended to serve as a manual of good practice for the design of prestressed concrete flat slabs.

In addition to the recommended procedures, other methods are described for the sake of completeness and to compare different methods of design.

The Report was produced by a sub-committee of the Joint Structural Division of the South African Institution of Civil Engineers, and the Institution of Structural Engineers.

The Committee consisted of:

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A. E. Goldstein wrote most of the text, drew the diagrams, and wrote the computer programs.

It should be noted that a decimal point has been used in the text. Computer output generally uses a point and not a comma, and it was felt that mixing two systems would be confusing.
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Appendix D  Neutral Axis depth and crackwidth at serviceability loads
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Appendix F  Shrinkage deflection
### Notation

- **A**: Area, Constant in Long's formula
- **a, b**: Sides of rectangle (a is smaller side)
- **A<sub>ps</sub>**: Area of prestressing tendon
- **A<sub>s</sub>**: Area of bonded reinforcement
- **a<sub>1</sub>, a<sub>2</sub>**: Distance from support to point of change of curvature in tendon
- **a<sub>cr</sub>**: Distance from surface of reinforcement bar to position of crack (Crackwidth formula)
- **b<sub>1</sub>, b<sub>2</sub>, c<sub>1</sub>, c<sub>2</sub>**: Dimensions of parabola (Appendix A)
- **C**: Torsion equivalent moment of inertia for stiffness
- **c<sub>1</sub>, c<sub>2</sub>**: Size of rectangular column capital in direction of span, and transverse (ACI col stiffness)
- **D**: Diameter of equivalent circular column capital
- **DL**: Dead Load
- **d**: Effective depth of slab (concrete to centroid of reinforcement)
- **E**: Young's Modulus
- **E<sub>c</sub>**: Young's modulus for concrete
- **E<sub>s</sub>**: Young's modulus for steel
- **f**: Stress
- **f<sub>c</sub>**: Concrete stress
- **f<sub>ct</sub>**: Tensile stress in concrete
- **f<sub>cu</sub>**: Characteristic stress of concrete
- **f<sub>ci</sub>**: Initial cube strength
- **f<sub>c</sub>**: Steel stress
- **f<sub>pc</sub>**: Equivalent tendon stress after losses
- **f<sub>pb</sub>**: Tendon stress at ultimate limit state
- **f<sub>0.1k</sub>**: Tendon stress at 0.1% strain
- **G**: Shear modulus
- **h**: Overall depth of section
- **I**: Moment of inertia
- **I<sub>g</sub>**: Gross moment of inertia
- **I<sub>e</sub>**: Effective moment of inertia
- **α**: Angle of shear reinforcement to horizontal
- **Δ<sub>in</sub>**: Increase in tendon stress due to bending deflection
- **γ**: Loss in prestress due to draw-in
- **I<sub>e</sub>**: Equivalent moment of inertia
- **J**: Lever arm factor
- **k<sub>eq</sub>**: 'Wobble factor' in friction eqn.
- **k<sub>c</sub>**: Equivalent column stiffness
- **k<sub>cl</sub>**: Lower column stiffness
- **k<sub>cu</sub>**: Upper column stiffness
- **K<sub>1</sub>, K<sub>2</sub>, K<sub>3</sub>, K<sub>4</sub>**: Coefficients in equation for span-depth ratio
- **L**: Span centre to centre of columns
- **L<sub>1</sub>, L<sub>2</sub>, LL**: Live load
- **l<sub>i</sub>**: Span in direction moments are calculated
- **l<sub>2</sub>**: Span in transverse direction
- **m**: Loss of prestress/unit length
- **M**: Moment
- **M<sub>cr</sub>**: Moment at first cracking
- **M<sub>0</sub>**: Design moment at midspan
- **M<sub>x</sub>**: Moment in x direction
- **M<sub>y</sub>**: Torsional moment
- **M<sub>eff</sub>**: Effective moment in x direction
- **M<sub>d</sub>**: Design moment at midspan
- **M<sub>x</sub>**: Moment in x direction
- **M<sub>y</sub>**: Moment in y direction
- **M<sub>xy</sub>**: Torsional moment
- **M<sub>x</sub>**: Effective moment in x direction
- **M<sub>y</sub>**: Effective moment in y direction
- **MR**: Moment of resistance
- **P**: Prestress
- **PF**: Final prestress
- **PL**: Permanent load
- **P<sub>0</sub>**: Effective prestress at distance x
- **P<sub>x</sub>**: Effective prestress at distance x = 0 form anchorage
- **Q**: Proportion of permanent load carried by prestressing shear perimeter
- **u**: shear stress
- **v**: Permissible shear stress
- **W**: Load
- **x**: Depth of neutral axis
- **Z**: Section modulus
- **ε<sub>m</sub>**: Effective strain in tension at outer fibre
- **ρ<sub>pr</sub>**: Percentage of bonded reinforcement
- **ρ<sub>ps</sub>**: Percentage of prestressing steel
1.0 Introduction

In 1989 the Structural Division of the South African Institution of Civil Engineers created a sub-committee to examine the design of prestressed concrete flat slabs. It was found that a certain amount of poor design was prevalent, and the committee decided to produce a booklet of recommendations for good practice.

The matter was considered especially important because the South African Loading Code was changed with effect from 1990, and the required factor on D.L. is now 1.2, whereas it was previously 1.4. This has the effect of reducing reinforcement areas, and cracking and deflection require more attention. To make allowance for this, SABS 0100 was revised, and among other changes, the allowable concrete shear stress was reduced by 10 percent, to lessen the probability of brittle shear failures.

1.1 Flat Slabs

Flat slabs were originally invented in the USA at the beginning of this century, and there were a number of patented systems. The early reinforced concrete flat slabs all had drops, and columns with capitals, and were considered to be the structure of choice for warehouse construction and heavy loads. Because of the columns capitals and drops, shear was not really a problem. Design was based on tests on stresses in reinforcement at working loads, and the early codes required a total moment in a span of $WL^2/11$.

It was realized that statically a total moment of about $WL^2/8$ was required for equilibrium, (If the column diameter is D, the statically required moment is (very closely) $W(L-2D/3)^2/8$ where $L-2D/3$ is the effective span. The difference between $WL^2/11$ and $WL^2/8$ was attributed to a mystical '2 way action'. In fact it was due partly to tensile stresses in the concrete and partly to arching effects reducing the measured stress in the reinforcement.

The philosophy, and the empirical coefficients, persisted until the 1950's when the allowable stresses in reinforcement were increased, limit state design was introduced, and the statically required moment of $WL^2/8$ was introduced into the codes. This was because it was felt that it was not safe to rely on arching or tensile strength of the concrete. In addition to the changed moment coefficients, the frame method of analysis was required in certain cases.

1.2 Flat Plates

Flat plates were subsequently developed, with no drops and no column capitals, and due to the much cheaper shuttering required, they became popular for residential and office buildings.

A number of catastrophic shear failures of flat plates occurred, including some where several floors of a building suffered progressive collapse. As a result a large amount of research into shear in flat slabs has taken place, and various methods of reinforcing slabs against shear failure have been developed. Because of the brittle nature of shear failures, conservative design is necessary.

1.3 Prestressed Flat Slabs and Plates

Prestressed flat slabs and plates have been developed mainly in Australia and the USA over the last 30 years. They have a number of outstanding advantages. Among these are a shallower depth (for the same deflection), quicker stripping of shuttering, and greater shear strengths than plain reinforced slabs of the same depth. Prestressing is also applied to waffle type slabs to achieve even greater spans.

Tendons in post-tensioned concrete are considered to be either bonded or unbonded. Unbonded tendons are usually single strands covered with grease and an outer plastic sheath. Bonded tendons usually consist of a number of strands in a metal sheath, which is grouted after the tendons are stressed. Unbonded tendons have the advantage of low friction values, maximum lever arm and drape due to the smaller diameter, fast placing and avoidance of grouting operations. In the USA and UK, and in South Africa prestressed flat slabs have been almost entirely unbonded, whereas in Australia bonded tendons are the rule.

The disadvantage of unbonded tendons is that the prestress depends on the anchorage remaining intact throughout the life of the structure. Corrosion or accidental damage could cause tendon failures at any time, and detailing must take account of this, to try and prevent or reduce the possible effects. The USA Uniform Building Code requires that for one-way slabs unprestressed reinforcement sufficient to carry the Dead load + 1/4 live load at ultimate, is provided to prevent catastrophic failures in the case of loss of prestress. The committee considers that prestressed flat slabs do not have a better record than one-way slabs, and its recommendation is given below.
2.0 Design Codes

In the UK and S.A. the first generally accepted 'codes' for the design of prestressed flat slabs were Technical Report 17 of the Concrete Society, published in 1979, and Technical Report 25, published in 1984. There have also been the CEB-FIP design code and other overseas codes which have not had much influence in S.A., except that a commercial computer program used in South Africa is based on the ACI code. Technical Reports 17 and 25 are based on the load balancing method for preliminary design, and on a frame method for analysis. The ACI code also uses the frame method but takes account of the torsional flexibility of the slab at the columns (See 5.4.5). which reduces the apparent column stiffness. The ACI code uses an effective span of L-D, instead of L-2D/3. Recently a certain amount of analysis has been done by the finite element method, and by grillage programs. These are discussed later.

If premature failure due to shear is prevented, the interior panels of flat slabs have considerable reserves of strength due to two way arching action, and membrane stress. There is no recognised design method which takes advantage of this at present, so it is not feasible to take the extra strength into account. It does, however, emphasize the fact that exterior panels are more vulnerable, and greater care must be taken with them. Because of the reserve of strength due to arching, the Committee recommends that the exterior and corner spans be designed with additional non-prestressed reinforcement, (similar to the USA Uniform Building Code for one-way slabs), but it is not considered necessary for internal spans.

The committee recommends that enough non-prestressed reinforcement be provided, so as to ensure that if 50% of the prestress in an external span is rendered ineffective for some reason, the span will still be able to support before failure, an unfactored ultimate load of DL + LL/4 or in the case of warehouses DL + LL/2). A reinforcement of 0.25% of

The 0.25% reinforcement should be concentrated largely in the column band, say 75% in the column band, and 25% in the slab band.

2.1 Prestress Level

Reports 17 and 25 require a minimum prestress level of 0.7MPa. The CEB-FIP code requires 1.0MPa and the ACI 0.86MPa. In a lecture given some years ago it was suggested that less than 1.4 MPa was too low a prestress, 1.4 to 3.5 MPa was the 'proper' range, and over 3.5 MPa would cause excessive shortening. Many slabs are now designed with stresses less than 0.7 MPa.

It is the Committee's philosophy that there is a continuous spectrum of concrete from plain reinforced to fully prestressed concrete, and that there should be no arbitrary limits on prestress levels. It is considered that prestressed flat slabs are essentially ordinary flat slabs, subjected to additional lateral loads from tendons, which reduce the effective load on the slab, and to longitudinal prestress which helps to reduce cracking. However if prestress levels appreciably lower than the ones recommended in Reports 17 and 25 are used, there is a greater onus on the designer to satisfy himself that the requirements for deflection and cracking are met. The main advantage of prestressing slabs is that the prestress acts as an upward load resisting the effects of dead load, reducing the long term dead load creep which is the cause of most of the deflection. For this reason it is considered that if less than half the dead load is balanced, a good deal of the advantage of prestressing is lost. (See section 3.2). However, the actual amount of prestress is an economic decision.

2.2 Design Method

1. Preliminary Manual Design
The method recommended depends on the load-balancing method made popular by Lin.
Prestressing is treated analytically by removing the prestressing tendon and replacing it by the equivalent forces that it applies to concrete. (Diag. 2.1).

By taking reverse curvature of the tendon into account, the complete prestress loading diagram on the slab is derived (Diag. 2.2).

The system may be described as follows:-

1. Decide on column centres, concrete grade (usually 30 or more) and preliminary slab thickness, as well as drops, capitals, etc. Choose a preliminary thickness from paragraph 3.2.2, and from shear considerations. The shear will often govern, unless capitals or drops are used, and should be checked first.

2. Decide on the amount of load to be balanced. (This is an economic decision)

3. Using the maximum drape possible, decide on the preliminary geometry of the tendon. From (2) and (3) calculate the prestress force required.

   For a parabolic tendon of drape h, length L, Prestress P, the equivalent upward load is given by \( W = \frac{PL^2}{8h} \) and if L and W and h are known, P may be calculated (see also 4.2).

4. Analyse the slab for Dead Load, Live Load (for pattern loading see 4.1) and prestress load (use a preliminary estimate for losses)

5. Calculate the working stresses at various points and check for allowable tensile stress and deflections. It should be noted that the limitation of tensile stress is not a good way to limit cracking, but is acceptable for preliminary design. Final design should be based on limiting crackwidth.

6. Calculate the reinforcement required and check the maximum compressive stress in the concrete for the ultimate limit state.

7. Check the shear stress, and calculate the reinforcement required, if any.

8. Depending on the results of (5) and (6) it may be necessary to adjust the thickness and prestress, and perhaps to supply additional non-prestressed reinforcement to control cracking or shear. Non-prestressed reinforcement is always required over columns, and in external spans.

9. Check the prestress losses due to shrinkage, creep, friction etc. and adjust the prestress loading.

   It should be noted that although the sample calculations take account of the variation in prestress along the length of the structure, and of the effect of curvature of cables over the supports, it is sufficient for preliminary design to assume a uniform prestress along the length of the structure (except where the prestress is varied by changing the number of tendons, and to assume that the cables hang from the supports i.e. there is only upward distributed load from the tendons, and downward point loads at the supports. (See Diag 3.4) In deciding whether to vary the number of tendons, it may be taken as a rule of thumb that 7m of tendon is equal to the cost of two anchorages.

   Some designers make the above assumptions even for final design. However it should be realised that one is designing the non-prestressed reinforcement for the difference between the downward loads due to permanent load and live load, and the upward load due to prestress. The difference between these quantities may be sensitive to small variations in either one.

2.3 Preliminary Design Using Computer programs.

   Several programs are available which enable changes in the slab thickness, concrete grade, number of tendons etc. to be
easily made, and the results noted. The programs show stress, deflections, and additional reinforcement required, as well as highlighting areas of excessive shear stress. The preliminary design may be very quickly established, and a final analysis performed as in 2.2.4 to 2.2.8 above. For final design using commercially available programs, see the comments in 5.0.

2.4 Summary of Design Process

The design method is iterative.
1. Establish or revise design
2. Analyse
3. If results are not satisfactory repeat (1) and (2).

If a good choice of parameters is made, only one or two iterations may be necessary. As the effective stresses and deflections depend on the difference between the applied loads and the prestress balancing load, there is an advantage in the design process to making the difference small. However the actual prestress load required depends on economics, and it may well be economical to make the slab a bit thicker, add more reinforcement, and reduce the prestress to obtain the cheapest design.

In a multi-storey building it is usually better to make the slab as shallow as possible to save on architectural cladding costs, as well as on column loads and foundations, and the greater part, or all, of the dead load should be balanced.

3.0 Preliminary Choice of Parameters

3.1 Loads

The design is dependant on the loads applied. The loads are usually fixed by the clients requirements, but the live load, finishes and partition loading must be carefully considered and provision made for possible future changes in use. An important aspect is loading during construction. In multi-storey work, the load on a slab depends on the cycle of casting and stripping props, and the total load during construction can exceed twice the self weight of the slab. This may be appreciably more than the dead + live load case, and the resultant tensions can cause cracking, and deflections can be built into the slab being cast even before it is stripped.

3.2 Depth

The design of a flat slab has to meet two major requirements, strength and deflection.

3.2.1 Strength

The slab must be deep enough that shear failure is prevented (See 8.0) and that the section taking moment is strong enough. (mostly only critical at external columns and at the first interior column). Shear failure may be prevented in 4 ways.
1) By making the slab deep enough
2) By increasing the shear perimeter by using columns with capitals, or larger columns
3) By increasing the slab depth locally - ie. by drops
4) By using shear reinforcement, which can consist of stirrups, inclined bars, or steel studs welded to a steel plate, or welded steel shear-heads, or by increasing the level of prestress or bending reinforcement. (Although the equations in the code show that shear strength is related to area of reinforcement, the effectiveness of using additional bending reinforcement on the shear strength is doubtful, and this last method is not recommended).

3.2.2 Choice of slab depth for deflection control

Common practice in South Africa is to use span-depth ratios in the range 28 for heavily loaded slabs, to 40 or 42 for lightly loaded slabs, such as slabs for domestic dwellings, or roofs.

Report 25 states:

<table>
<thead>
<tr>
<th>Type of Construction</th>
<th>Loading</th>
<th>Span:Depth Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flat Plates</td>
<td>Light</td>
<td>40 to 48</td>
</tr>
<tr>
<td></td>
<td>Normal</td>
<td>34 to 42</td>
</tr>
<tr>
<td></td>
<td>Heavy</td>
<td>28 to 32</td>
</tr>
<tr>
<td>Waffle Slabs</td>
<td>Heavy</td>
<td>28 to 32</td>
</tr>
</tbody>
</table>

Where the span-depth ratio relates to the longer span.
In this report a formula is proposed (in 3.3.8) and its derivation explained in Appendix B. A comparison of the formula with the nomogram in Report 25 is also given.

The proportion of LL which should be taken as permanent load depends on the type of loading. (see 4.1.5)

3.3 Deflection

It is necessary to decide on the allowable deflection, which depends on:
1) The presence of brittle partitions, walls and finishes.
2) Required limits to slopes for drainage or machinery
3) Aesthetic considerations
4) Requirements for vibration control, which is related to flexibility, and therefore to deflection.

3.3.1 Allowable Deflection

Allowable long term deflections are usually given, for normal buildings, as span/300 or 20mm, whichever is smaller, or span/500 if there are rigid partitions. Even span/500 may be unsafe if there are large panels of brittle partitions. In this case a value of span/750 may be appropriate. Various guidelines have been given by various authorities.

SABS 0160-1989, Table E-2:
Different limits are given, ranging from Span/300 to Span/500, depending on the limit of deformability. A limit of 10mm is given to prevent horizontal cracking in walls.

ACI 318-1989, Table 9.5(b):
Total deflection occurring after non-structural members are installed should not exceed span/480.

Concrete Society Technical Report 17:
Deflection, including the effects of creep, should not exceed Span/350 or 20mm in cases where partitions may be affected by deflections.

For spans over 9m, greater care should be taken.

For the calculation of deflection, see 6.3.1

3.3.2 Creep and Shrinkage

Creep

The major factor in deflection is the long term component, due to creep of the concrete under compressive stress. The amount of creep is affected by the humidity level, and in the very dry conditions which occur in much of S. Africa at times, the creep can be higher than in the UK or Europe. Some local aggregates give rise to very high creep, and figures of as much as 5 times the short term deflection have been recorded. Where aggregates are known or suspected to be liable to excessive creep, tests should be done.

Because long term creep deflection is important, prestressing which counterbalances an appreciable part or all of the dead load, is very effective in preventing excessive deflection.

The depth of the slab has to be carefully considered in relation to the amount of prestress, and where the prestress is low, the slab must be made relatively deeper to control deflection. Partially prestressed slabs are especially susceptible to long term creep because of the very early age at which shutters are struck; typically at 3 days, when the concrete reaches 15 MPa. (This can increase creep by up to 50%) (See BS8110 or SABS0100 part II). If deflection control is very important, consideration should be given to prestressing and stripping the slab at a later stage, e.g. when the strength reaches 25 MPa. This will reduce creep losses as well as deflection.

Long-term deflection can be determined by considering the ratio of long-term to short-term curvature of the member. The properties of aggregates can influence the deflections to a large extent. Methods are being developed for allowing for the effects of different aggregate types.

Shrinkage

Shrinkage can be a considerable factor influencing the deflection of reinforced concrete members. This is because eccentric bonded reinforcement restrains the shrinkage, causing warping and deflection. The amount of deflection is related to the area of reinforcement, and for prestressed flat slabs with unbonded tendons, which only have appreciable reinforcement over columns, the effect is considered likely to be small. The effect is greater for cracked slabs, and if cracking is likely, should be considered. It may be analysed by conceptually separating the reinforcement from the concrete, allowing the concrete to shrink, applying compressive forces to the reinforcement and placing the reinforcement back in the concrete. Forces equal and opposite to the compressive forces are applied to the complete structure. Alternatively the shrinkage can be simulated by temperature stresses. Creep has to be allowed for. A sample calculation is given in Appendix F.
3.3.3 Shrinkage Cracking

Where the distance between expansion joints is large, shrinkage stresses may cause additional cracking, which further reduces the stiffness of the slab. Some South African aggregates have very large shrinkage, and if small deflections are essential, tests should be done.

Both temperature and shrinkage effects are worse where there are stiff columns or walls, and special precautions may have to be taken (e.g. leaving gaps to be cast later.) See Diag. 3.1

Diagram 3.1
3.3.4 Live Load
Because deflections due to live load may be important, although there is no appreciable creep component for non-permanent live load, slabs also have to be thicker where the live load is high, (in addition to problems relating to shear and compressive stresses due to bending)

3.3.5 Internal and External Spans
Because of the reduction of continuity at the outer column, moments in outer spans tend to be greater, and so, of course, are the deflections.
It is therefore desirable that the external spans should be shorter than the internal ones. (Preferably about 10% to 20% shorter). If this is not possible, the slab depth may be increased in the outer spans and if a uniform slab depth is desired, in the internal spans as well. Alternatively a greater proportion of load can be balanced in the external span. This is usually the preferred solution.

3.3.6 Temperature Stress.
Where the slab is exposed directly to the sun, temperature stresses may become sufficiently large to crack the slab, even if there is a fairly high level of prestress. This reduces the effective stiffness of the slab, and increases deflection. Insulation is strongly recommended

3.3.7 Grade of Concrete
Because the Modulus of Elasticity (E) of the slab directly affects deflection, and because the Modulus of Elasticity is related to the grade of concrete, this factor needs to be carefully considered.
Although SABS 0100 gives the average relation between normal concrete grade and Young's Modulus as $E = (20+0.2Fc)\text{GPa}$ it should be noted that $E$ can vary considerably, and if deflections are critical, tests of the concrete should be made, or the paper by Dr. Mark Alexander giving $E$'s for S.A. aggregates should be consulted. (The Civil Engineer in South Africa Vol 27, No 6. June 1985). As actual $E$'s can vary considerably, if the aggregates are not controlled, a value of 0.8 of the above formula value is recommended.

3.3.8 Formula for Choice of slab depth.
To help in the preliminary section of slab depth, a semi-empirical formula taking the above factors into account has been derived.
The formula is:

$$\frac{L}{h} = \left[ 1.4 + \frac{53}{(P_L \times 3.5 + L_L - QP_L \times 3.32) \frac{1}{3}} \right] \cdot K_1 \cdot K_2 \cdot K_3 \cdot K_4$$

where $P_L$ is Permanent load in KPa ($3.5$ is a creep factor)
$L$ is Live Load in KPa
$Q$ is proportion of Permanent Load carried by prestress. (The factor of $3.5$ for creep is multiplied by $0.9$ to allow for possible losses)

$K_1$ is factor for end span or internal span: 
$K_1 = 0.90$ for End span
$K_1 = 1.0$ for Internal span

$K_2$ is factor for cracking by temperature stress or shrinkage
$K_2 = 0.95$ if cracking is likely:
$K_2 = 1.0$ if it is not

$K_3 = (\text{E conc}/26)^{1/3}$
Where E conc is the expected short term E of the concrete to be used
(26 GPa is the expected E of 30 grade normal concrete)

$K_4$ is factor for drops or flat plates:
for flat plates 
$K_4 = 1.0$
for slab with adequate drops 
$K_4 = 1.15$
With sufficient experience the designer may decide to make \( K_1 = 0.95 \) for external spans, and 1.05 for internal spans.

### 3.4 Prestress Level

The minimum prestress requirements of the codes are partly to ensure that the concrete remains mainly uncracked, which reduces deflection. However in certain circumstances these suggested prestress levels may not be enough for this purpose, especially where temperature stresses are large (eg. in a roof slab exposed to the sun without insulation), or where shrinkage stresses are large.

It is not a recommendation of the Committee that any minimum prestress level is maintained, but if the prestress is less than 0.7 MPa, greater care must be taken to ensure that deflections and cracking are not excessive (ie. the serviceability limit states).

It is also considered that if the amount of load balanced by the prestress is less than half the dead load, there is not much advantage in prestressing the slab.

### 3.5 Tendon Profile and Layouts

There are a number of different possible tendon layouts in plan. The one favoured by the early designers in PSC flat slabs was to concentrate some in the column band, and spread the rest out in the slab band, in the same proportion as the reinforcement in traditional R.C. flat slabs ie. 60 to 75% in the column band, and 40 to 25% in the slab band. (Diag. 3.2)

This has to a considerable extent been replaced by a system where the tendons are concentrated over the columns in one direction, and spread uniformly in the other direction, as if the slab were spanning onto beams spanning between columns. (Diag. 3.3) This system gives the maximum effective drape, and the most effective use of prestress. Because there are fewer tendons over the columns in one direction, the shear strength may be somewhat reduced. If the column spacing is different in the two directions, the banded tendons would normally lie in the direction of the shorter span.

A system has also been developed where all the tendons in each direction are concentrated over the columns. This system has disadvantages, because the drape of the tendons in one direction has to be appreciably less than in the other direction in order that the tendons in one direction may pass over the tendons in the other direction at the columns. (Alternatively, the tendons could be 'woven' so that some of the tendons in one band pass over, and some under, the other band. This is not considered practical). In addition, the self weight of the slab is not uniformly balanced. The system does, however, give a better shear capacity. Reinforcement should be supplied in the 'slab' area between the bands to control cracking, and local shear failures due to concentrated loads.
It is also possible to provide prestressing in one direction only, and use ordinary reinforcement in the other direction. This would normally only be done if the spans in the two directions were very unequal.

Tendons are usually arranged in profile to obtain the maximum drape, and are normally fixed to approximately parabolic profiles, to give a fairly uniform upward load on the slab. (See Dia. 3.4). This maximum drape may be reduced in shorter spans, to keep the prestress level as uniform as possible. In external spans, the tendons at the outside edge are usually kept fairly close to the centreline of the slab to reduce problems with bursting. It is advantageous to lift them a small amount above the centreline to counteract the hogging moments, and to increase the drape as much as possible, which also reduces the sagging moments in the first span. Instead of parabolic profiles, more or less straight lines have also been used and in this case the uniform downward load of the slab is notionally balanced against upward point loads. The geometry is easier, and fewer stools are required.

4.0 Loading

4.1 Vertical Loads

4.1.1 Load Factors
The revised loading code, SABS0160, which was issued in May 1990, radically changed the required load factors and loading pattern.
The load factor for live load remains at 1.6 but that for dead load is 1.2 with a proviso that design moments for factored (Live + Dead) shall not be less than the moments for 1.5 DL.

4.1.2 Pattern Loading
In addition, pattern loading is not required for dead load, (which eliminates some of the discrepancy in flat slab design between the tables of coefficients, and accurate analysis).
It is therefore necessary, for flat slabs to consider 3 loading cases, which are calculated for working loads and then factored for ultimate load. (See Dia. 4.1). These are: Dead load on all spans, live load on even spans, and live load on odd spans. To these should be added the analysis for the prestress loading, making four analyses necessary.
These may be factored and combined for ultimate load limit states, as well as the limit states of deflection and cracking. Dead load x 1.2 + pattern live loading x 1.6 (or Dead load x 1.5) gives maximum sagging moments in alternate spans, and Dead load x 1.2 + uniform live load x 1.6 (or Dead load x 1.5) gives maximum hogging moments.

4.1.3 Load Factors for Serviceability State.

It is a requirement of the Loading Code that the dead load be factored by 1.1 (if 1.1 is more unfavourable than 1.0) for the states of deflection and cracking, as experience and tests have shown that the actual dead load of a structure is generally greater than would be expected from the nominal sizes. The Load Factor for live load would normally be 1.0.

4.1.4 External Spans

It is, strictly speaking, necessary that the effects of two-way action be taken into account eg. in the first bay of a multiple bay structure, the first internal strip tends to take more load than the average strip. A preliminary analysis may be done in the lateral direction. (Diagram 4.2).

The panel spanning along column line (2) & (4) will have additional load due to the greater negative moments at these points. This is often not taken into account in calculations, as it appears that designers feel that plastic redistribution will ensure that the slab is safe. Where shear stresses are high, and a brittle failure is therefore more likely, it should however be taken into account.

4.1.5 Permanent Live Load

The proportion of Live load which is taken as permanent varies with the type of loading, and may vary from 0% for garages, to 50% to 75% for storage facilities.

4.2 Lateral Load

Flat slab structures are occasionally designed as frames to take lateral loads due to wind. They are not well adapted to do so, because of the difficulty of transferring moments from the slab to the columns. Prestressed flat slabs are not suitable for earthquake loading because of reversal of stresses. Where earthquake loading is a requirement, other means of resisting the loading should be provided - usually concrete shear walls, or concrete frames with brick infill.

The frame to be analysed for lateral load is usually, in the British method, taken to have the stiffness of half the width of the panel, to allow for the effects of torsional flexibility (See 5.4.2). An appropriate 3 dimensional analysis (3d frame or finite element) will give good results.

4.3 Temperature Stress, and Shrinkage
If temperature stresses and shrinkage are important, the differential temperature between the top and bottom of the slab must be assessed. It depends on the exposure to the sun, the latitude and altitude, and the ventilation in the building. Shrinkage may be assessed from SABS0100, and converted for analysis purposes into a general temperature drop. Many frame and grillage programs allow thermal loadings, although Grillage programs would normally only allow for bending due to differential temperature, and therefore would not allow for shrinkage. Shrinkage will be important in long structures and where columns are stiff or shear walls exist, and may be analysed by simulating the shrinkage by a temperature drop in the frame analysis. (See Appendix F). Temperature stresses in South Africa tend to be higher than in Europe, and can cause large cracks, even where the analysis appears to show that there is no tension.

5.0 Calculation Methods

In addition to the methods given below, computer programs are available to do complete designs. The designer should check the assumptions carefully. It has been noted that some designers use appreciably lower prestress losses than would be calculated in terms of the code recommendations. In addition, the load and material factors should be checked. It is not acceptable to use ACI stresses with SABS load factors.

5.1 Load Balancing

This has already been mentioned in 2.2.1. If all the load were exactly balanced by the prestress, then there would be no resultant stresses, except compression in the concrete, and the slab would remain perfectly level. Obviously this is not possible (because the live load can vary), and the out of balance loading must be analysed by one of the methods below.

5.2 Coefficients

All codes give a series of coefficient for designing flat slabs by the empirical method. They may be used for preliminary design, but are not considered adequate for final design.

5.3 Yield Line

Yield line methods have been used to design reinforced concrete flat slabs, but because they do not take compatibility into account, problems of cracking and deflection have arisen. The methods are not considered suitable for prestressed flat slabs for serviceability conditions, and for ultimate load conditions, the rotation capacity required may not be obtainable.

5.4 Equivalent Frame

All codes allow the use of an equivalent 2 dimensional frame to analyse the effects of vertical and horizontal loads on flat slabs. A lot of experience has been accumulated with this method, and the codes are basically written with the frame analysis in mind.
The equivalent width of frame is taken as the distance between the centre lines of slab bands. The equivalent I (Moment of Inertia) of the slab is usually taken as the I for an uncracked slab. (But see later for corrections to this.) Drops should be taken into account if they exceed 1/3 of the slab width. This is rather tedious to do if hand calculations are used, but computer programs will handle them easily.

If cracking is likely, i.e., for low prestress, high temperature stress, and high shrinkage stress, the equivalent I should be reduced to that of a cracked section (Diagram 5.1).

Column stiffness must be taken into account, including the effects of capitals. Loading is taken to be appropriate to the width of the frame, although at the first internal frame it may be appreciably higher. (see 4.1.4)

5.4.2 Lateral Loads

If the columns are considered to be rigidly connected, as in the method of Report 25, the equivalent frame width is taken as one half that for vertical loads, due mainly to the ineffective torsional connection of the slab to the column. The correct column stiffness is important.

5.4.3 Waffle Slabs

Waffle slabs have a solid section adjacent to the column, and ribs and slab construction between column capitals. Because there is... less concrete in the ribs, there is more likelihood that a waffle... slab will crack and the section properties of the ribs should be based on a cracked section.

It is necessary to allow for the greater equivalent I in the solid section, and to take account of this in the analysis. The result is that the hogging moments over the columns are increased, and the sagging moments in the ribs reduced in comparison with a homogeneous slab.

The lateral distribution of moments between column and slab bands is similar to a solid flat slab except:

1. If the solid section is at least one third of the smaller dimension of the surrounding panels, the column band width should be based on the width of solid section.

2. The design moments to be resisted by the middle strip should be increased in proportion to its increased width, and the design moment in the column strip decreased by the same amount. It should be noted that although the total...
moment in the column band is reduced, the moment per unit width in the column strip is increased. In order to ensure that the normal punching shear clauses for flat slabs can be applied to the area adjacent to the column, the solid section should extend at least 2.5 times the slab thickness from the column face. Where a shear perimeter falls outside the solid section, the shear force may be considered to be distributed equally between the ribs, but where two ribs meet at a corner, the effective width of rib is only 1.4 times the rib width. (see Diag. 8.1). If shear reinforcement is required in the ribs, it should extend an effective depth into the solid section.

5.4.4 Young's Modulus

It is necessary to take two moduli of elasticity into account when calculating deflections, a short term modulus for live load, and a long term modulus for dead load, which takes into account the effect of creep. The one may be greater than the other by a factor of 3 or more. Because our climate is much drier than Europe or most of North America, the creep coefficients used in those countries are too low for our conditions. SABS0100 part 2 gives tables and graphs for estimating creep. (See also note in 3.3.7 re aggregates). Also the Young's modulus at initial prestress may be less because of the lower strength at that time.

5.4.5 Column Stiffness

BS8110 and SABS0100, and the ACI code treat the subject of column stiffness quite differently. The first two assume that the column is rigidly fixed to the slab over the whole width of the panel. The ultimate negative moment at the outer columns is checked, and if it exceeds the moment of resistance of the width of slab immediately adjacent to the column (See diag. 5.2) then the moment at the outer column must be reduced. It is not clear if it is intended that the moments at the first interior column must be increased, but the slab sagging moments must be increased to maintain equilibrium The ACI code, on the other hand, makes allowance for the loss of stiffness due to torsion (See Diag. 5.3) and reduces the column stiffness accordingly.. The ACI method is far more logical but Long (see below) gives a method which is simpler to use than the A.C.I. method. If a grillage or finite element program is used, the rather tedious ACI formula is unnecessary.
Because the BS 8110 method tends to overestimate the column moments, and because the BS8110 method requires that the moment at the outer columns be reduced if it exceeds the moment of resistance, it is desirable to model the column stiffness more accurately.

It is recommended, if the frame method is used, (and not grillage or finite element analysis), that the Long or the ACI method of calculating column stiffness allowing for torsional stiffness of the slab be used. (See diag. 5.4)

The torsional stiffness of a rectangular section is given by \( C_G \) where \( C \) is the equivalent torsional moment of inertia, and \( G \) is the shear modulus, or approximately \( G = 0.4 \, E \) (Young's Modulus)

\[
C = 1/3 - (3.36a/16b)(1 - (a/b)^4/12))a^3 b
\]

where \( a \) is the smaller dimension of the rectangle, and \( b \) the larger. A simpler but less accurate formula is:

\[
C = 3 \, b^3 d^3/(10(b^2 + d^2))
\]

Where the section is composed of several rectangles, the torsional moment of inertia may be estimated by adding the \( C \)'s for the individual rectangles.

ACI 318 states 'the stiffness \( k_t \) of the torsional members shall be calculated by the following expression'

\[
k_t = \sum (9 \, E_c C/(l_x(1-c/l_x)^3))
\]

where \( c \) and \( l_x \) relates to the transverse spans on either side - (see diag 5.3) and \( E_c \) is the Young's modulus of the concrete slab.

It is common practice in grillage analysis of bridge decks to reduce the torsional stiffness of members by one half, to allow for the effects of cracking, and this is recommended.

The stiffness of the columns above and below should be calculated taking account of the capitals and increased inertia at slab level. However, if these are neglected, the column stiffnesses will be approximately \( 4EI/L \)

then \( 1/K_{eq} = 1/K_{cu} + 1/K_{cl} + 1/K_t \)

Where \( K_{eq} \) is the equivalent column stiffness

\( K_{cu} \) is the upper column stiffness

\( K_{cl} \) is the lower column stiffness

\( K_t \) is the torsional stiffness

---

**Diagram 5.3**
The stiffness of the slab may be calculated by assuming that the gross section of the concrete is effective.

$$I = b d^3/12$$

A more accurate method is to use the equivalent I value based on the cracked section, but this requires an iterative analysis.

Using these values a structural analysis can be made.

Long's method of calculating the effective stiffness of the column is:

If the span is L, the column dimension in the direction of the span is c, the column stiffness is $k_c$ (4EI/L for a prismatic section), the slab (or beam) thickness is h then the equivalent column stiffness is

$$k_e = k_c/(1+A k_c L/(E h^3 c))$$

where A is .0564 for interior columns and .1272 for exterior columns.

If the column has capitals, or drops are used, they should be taken into account in the column stiffness.

Long's method gives a slightly larger stiffness than the ACI method for internal columns.

The design moments are taken at the face of the columns, and at about mid span, but the sum of the design moments must not be less than $W (L-2D/3)^2/8$. If there are column capitals, the critical section for moment may not be at the face of the column, but somewhat away from the face, where the effective depth is less, and the tendon is lower. This only applies if the increased thickness of the capital is taken into account for calculating the slab strength, which is not usually done.

5.4.6 Stiffness with drop panels
First calculate the I of the cross section with the drop.

Two methods are common.
1) To calculate as a T beam, assuming that the drop is sufficiently wide to force the slab to act with it (see Diag 5.5.1).
2) To calculate the separate I's and add them as if the section were Diag. 5.5.2

Then the effective stiffness of the slab in the longitudinal direction can be obtained from the tables in the ACI code, from Column Analogy, or from a structural analysis program.

5.5 Grillage Analysis
5.5.1 General
Because grillage analysis programs are fairly generally available, a grillage analysis is an attractive method for final analysis after the preliminary design has been done, especially if the slab is irregular and does not fall in the range for which the frame analysis was developed.

In "Design of reinforced concrete flat slabs, to BS8110" - CIRIA report 110, Robin Whittle has written a chapter on grillage modelling, and has favourably compared the results of the analysis with experimental results.

The member layout recommended is shown in Diags. 5.4 and 5.7. It should be noted that if the grid is too coarse, the total calculated moments in each direction will be somewhat less than the statically required moment. The effect for the recommended spacing is negligible.

A full frame in one direction should be analysed, with preferably 2 or 3 bays in the other direction. For an internal bay, points of maximum moment and zero shear may be assumed at the centre lines of slab bands. Pattern loading should be taken into account.

5.5.2 Summary of grillage analysis.
The method may be summarised as:

5.5.2.1 The layout is based on centre lines of columns. Columns are represented by point supports (with bending stiffness in 2 directions).
5.5.2.2 A member is used to connect columns (A1, A2 Diag. 5.4).
5.5.2.3 The stiffness of these members is based on a width equal to the width of the column plus a slab depth. This ensures that the local nature of moment transfer is realistically modelled, because this is where the stress concentration exists.
5.5.2.4 Lines of members should connect centre lines of panels (D1, D2)
5.5.2.5 Lines of members should be placed at quarter points of panels (C1, C2).

5.5.2.6 A line of members should be placed at about the width of members A1, A2 from the centre lines of columns. If spans are small, this line may be omitted.

Diagram 5.4

5.5.2.7 Member width should be based on half the distance between centre lines of members (see Diagram 5.6).

5.5.3 Loading
Whittle states that member loading gives better results than nodal loading. The recommended method of distributing slab loads is shown in diag. 5.6 but a uniform loading may be assigned in proportion to the ratio of the sides - (eg. for a panel with an aspect ratio of 2, the UDL is 1/3 of the total load on each long side, and 1/6 of the total load on the short side.) For sides of a and b, the loading would be a/(2(a+b)) on side a, and b/(2(a+b)) on side b.

5.5.4 Column Stiffness
Columns are normally considered fixed at the remote end, and the equivalent stiffness of the column support is 4EI/L, but if there are capitals they should be taken into account.
If the remote end is supported on a small footing, 3EI/L is more appropriate. Because the torsional stiffness of the slab is taken into account in the grillage method, it is not necessary to reduce the column stiffness as in the ACI frame method.

5.5.5 Choice of member properties
Separate analyses are required if the serviceability and ultimate limit states are to be accurately modelled, as the state of cracking is likely to be different. If an accurate analysis is required, an iterative process is required. First moments and stresses are calculated, then the state of cracking is assessed, and the analysis is repeated.
For the cracked section properties, it may be assumed that the tensile stress in the concrete is 1.0 MPa for short term deflections, and 0.55 MPa for long term. The analysis to obtain the moment of inertia is complicated, but can be handled...
by a computer program (See Appendix D). A reasonable simplification is to calculate the neutral axis, assuming no tension in the concrete, and calculate the reinforcement stress. The additional moment of inertia from tension stiffening is given by Whittle as b(h-x)^3 f_t E_s/(3 f_c E_c) (see Diag. 5.1). In the case of prestressed slabs the position of the neutral axis may be calculated by moving the line of the compressive force to the line of the reinforcement, and increasing the moment. (See Appendix D)

For approximate analysis, which is normally adequate, it may be assumed that the gross moment of inertia of the uncracked slab (ignoring reinforcement) reduces by 1/2 when cracked.

5.5.6 Torsional properties.
Cracking of the edge beam reduces its torsional stiffness near the column.
BS8110 recommends that the torsional constant of the beam be taken as 1/2 the St. Venant Value.
\[ C = 1/2 \frac{k a^2 b}{k} \]
where \( a \) is the smaller dimension, \( b \) is the larger, and
\[ k = \frac{1}{3} - \frac{(3.36a / 16b)(1 - (a/b)^4/12)}{2} \]
G may be taken as 0.4 \( E_s \).

Torsional moments may be combined with bending moments by the method of Wood and Armer. (Concrete 1968,2, pp. 69-76) i.e. by taking an effective moment \( M_{\text{eff}} = M_x + K M_y \) in the x direction, and similarly, \( M_{\text{eff}} = M_y + M_x(1/K) \) where \( K \) is normally 1.0 but may be chosen by the designer.

5.5.7 Waffle Slabs
Torsional stiffness may be neglected in the ribs, but should be taken into account in the solid area.

5.5.8 Grillage - Transverse Loads
A grillage is not applicable for analysing lateral loads, but a 3 dimensional frame program, using the same modelling as the grillage, with vertical members for columns, will give good results.

5.6 Finite Elements
Elastic finite elements are not really well adapted to analysing flat slabs. The time taken is longer than for an equivalent grillage program, it is more difficult to adjust the stiffnesses to allow for cracking, and the results are not easy to interpret.

Most grillage and finite element programs suffer from the disability that it is difficult to model a finite size of column, and the elastic solution for slabs supported on a point support gives an infinite moment at the point support. Consequently the smaller the grid or finite element mesh, the larger will be the local moments given by the program. Non-linear finite elements do not have this disadvantage, but it is not yet feasible to use non-linear finite elements for routine office design. The use of a member on the column centreline slightly wider than the column gives good results in a grillage analysis. If finite elements are used, the elements should follow the recommendations for grillage analysis in 5.6.3, and column stiffness must be taken into account, and a complete frame in one direction should be modelled, with, preferably, at least 2 or 3 bays in the other direction. The resultant torsion is dealt with in the same way as for grillages. (See 5.5.6)

5.7 Geometry of tendons

Report 25 gives a method for calculating the tendon geometry. (Given here in Appendix A). It is necessary to assume values for \( a_1 \), \( a_2 \), and for the dimensions from the soffit of the slab \( b_1 \), \( b_2 \), \( b_3 \).

It is common to assume \( a_1 \), \( a_2 \) to be 5% of the span, but it is more logical to proportion them so that the radius of the tendon over the column is a reasonable value considering the tendon diameter - say 100 tendon diameters, and assume the cap is actually a circle, not a parabola. There is some advantage to having the maximum slope at the critical shear perimeter as the inclined tendon force reduces the shear stress. Some designers use a constant value of \( a_1 \), \( a_2 \), throughout, even with different spans.

The equivalent upwards and downward loads applied by parabolic tendons to the slab are given by \( PH = wL^2/8 \) where \( H \) is the sag and \( L \) the span. (See Diagram in Appendix A)
\[ \text{ie. for A-B,} \quad w = 8P c_1/(2a1)^2 = 2P c_1/ a_1^2 \text{ downward} \]
\[ \text{for BCD, (Sag of left side) = (b_1-c_1-b_2), and equivalent span = 2(X-a_1)} \]
and equivalent \[ w = 8P(b_2-c_1-b_2)/(2(X-a_1))^2 \text{ upwards} \]

5.8 Friction and other losses

Losses in prestress need to be estimated initially, and checked when the design has crystallized.
Losses to be calculated are:
1. Due to friction
2. Due to 'draw-in' at the anchorage.
3. Due to elastic shortening. This loss could be reduced by re-stressing the tendon, but this is hardly ever done in slabs.
The loss is about 0.5% of the initial prestress, and is sometimes neglected in practice.

4. Due to shrinkage

5. Due to creep -- creep losses may be reduced by stressing at as late a date as possible. Creep is one of the larger losses.

6. Due to relaxation of the prestressing steel

Relaxation information should be obtained from the manufacturer. SABS 0100 states that the loss should not be less than the 1000 hour relaxation figure. (BS8110 gives multipliers varying from 1.5 to 2.0 on the 1000 hour figures). If the manufacturers figures are not available, a loss varying between 10% at an initial prestress of 80% to 3% for an initial prestress of 50% of characteristic strength may be assumed. Low-relaxation strand normally available, losses of 3% to 4% are commonly used.

Adequate information about items 1 to 5 is given in the codes.

Loss due to friction may be split into 2 parts, one due to 'wobble' in the sheath, which occurs even if the tendon is straight, and one due to curvature of the tendon. 'Wobble' may be attributed to the tendons sagging between supports, and thus causing additional curvature.

The effective prestress at any distance x immediately after stressing, and before the anchorage is set and 'draw-in' occurs is given by

\[ P_x = P_0 e^{-kx \theta} \]

where \( k \) is the 'wobble factor', \( x \) is the length, \( \theta \) is the total angle that the tendon has turned through in radians, and \( \mu \) is the friction factor.

For the unbonded tendons used locally, the value for \( k \) is often taken as .0025. (see SABS 0100). \( \mu \) is sometimes taken as 0.12 for the type of tendon and sheath used locally, although CP110 gives 0.25. Report 17 gives 0.12.

FIP recommends \( \mu = 0.05 \), and \( k = 0.01 \), which is considerably less, but is justified for greased tendons. It is noted that some prestressing suppliers use \( k = 0.0025 \), and \( \mu = 0.06 \), based on tests.

The committee recommends \( k = 0.001 \) and \( \mu = 0.06 \) for strand available locally.

The angle turned through may be calculated by the simple geometry of the parabola, since the tangent to a parabola passes through a point equal to the drape of the parabola below the mid point.

In the diag. in Appendix A (From Fig. 73 report 25) the angle \( \theta \) at mid span is given by

\[ \arctan \left( \frac{2 c_1 / a_1}{x-a_1} \right) + \arctan \left( \frac{2 (b_1 - b_2 - c_1)/(x-a_1))} \right) \]

An alternative way to calculate friction losses is to calculate the lateral pressure on the concrete due to the tendon curvature (this is normally done to calculate the moments due to prestress), and multiply the lateral pressure by the coefficient of friction. (See 'Friction losses ...by equivalent load method. E. Keyder, PCI Journal March-April 1990). It is difficult to take account of 'wobble' by this method unless an equivalent angle is added to allow for it.

The loss due to 'draw-in' is calculated on the assumption that the loss of prestress due to friction is a straight line. (See example in report 25, p36).

If the 'wedge set' or 'draw-in' is \( x \) mm, (usually 4 to 8mm) then it is assumed that friction acts in the opposite direction and if the prestress loss is \( \gamma \), and the length over which the loss occurs is \( L \), then if the prestress loss/metre is \( m \) (kN/m)

\[ \gamma = \frac{L}{2AE} \]

but

\[ \gamma = 2mL \]

Therefore

\[ mL^2/AE = x \]

and

\[ L = \sqrt{AEx/m} \]

then

\[ P_f = P_o - 2mL \]

i.e. knowing \( x, m \), calculate \( L \), then knowing \( P_o \) calculate \( P_f \) which gives the final prestress after draw-in losses, but before losses due to shrinkage and creep.

From this the other losses are deducted to give the final prestress. For short spans the wedge-set effect is important. With greased tendons, there is a tendency for the stress in the tendon to even out over a period of time, but it is considered that a low values for friction takes account of this.
6.0 Serviceability Limit States

6.1 Allowable stresses at Serviceability Limit State

It is considered that stresses are not a serviceability limit state. (Tensile stresses calculated on un-cracked sections do not correlate well with cracking), but it has been traditional to calculate tensile stresses to limit cracking. For this reason the method is included, but the committee recommends that cracking is controlled by incremental stress and that the tensile stress method is used for preliminary design only.

It should be realised that shrinkage and temperature stresses tend to be higher in South Africa than in Europe, and cracking is likely to be more severe. However tensile stress is a good indicator of where cracking may be a problem.

The stresses in the concrete at serviceability limit state can be calculated by the formula

\[ f_c = \frac{P}{A} \pm \frac{M}{Z} \]  

(assuming an uncracked section)

where M is the nett moment from the analysis for dead load, live load, and balance load due to prestress at the critical section - either the face of the column or near mid span.

The tensile stress is limited, to reduce cracking, to the values given in the table (from Report 17). These vary from \( 0.15 \sqrt{f_c} \) to \( 0.45 \sqrt{f_c} \).

Since the tensile strength at transfer (stressing) may be less, this is taken into account. The formula really only applies to an uncracked section, and even if the tensile stress is limited, there is no guarantee that the section will not crack, due e.g. to shrinkage or temperature stresses.

As bonded reinforcement will always be supplied over columns, the third column is applicable. Cracking stress in concrete is usually about \( 0.9 \sqrt{f_c} \) MPa but varies considerably.

Compression stresses are not related to a serviceability condition.

Presumably the amount of bonded reinforcement which qualifies the section to qualify for column 3 is not less than 0.15% of the section area, the minimum quantity recommended in Reports 17 and 25.

<table>
<thead>
<tr>
<th>LOADING CONDITION</th>
<th>PERMISSIBLE COMPRESSION</th>
<th>PERMISSIBLE TENSION</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>With Bonded Reinforcement</td>
<td>Without Bonded Reinforcement</td>
</tr>
<tr>
<td><strong>Maximum Stress at Transfer</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>At sagging moment locations</td>
<td>( 0.33 f_c )</td>
<td>( 0.45 \sqrt{f_{ou}} )</td>
</tr>
<tr>
<td>At hogging moment locations</td>
<td>( 0.24 f_c )</td>
<td>( 0.45 \sqrt{f_{ou}} )</td>
</tr>
<tr>
<td><strong>Maximum Stress at Service</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>At sagging moment locations</td>
<td>( 0.33 f_{us} )</td>
<td>( 0.45 \sqrt{f_{us}} )</td>
</tr>
<tr>
<td>At hogging moment locations</td>
<td>( 0.24 f_{us} )</td>
<td>( 0.45 \sqrt{f_{us}} )</td>
</tr>
</tbody>
</table>

If the concrete is subject to severe temperature stress (e.g. due to sun) or to shrinkage stress, these values may not be adequate, and either the prestress should be increased, or extra bonded reinforcement should be supplied.

A better method is to calculate the incremental stress in the reinforcement, and limit it to some arbitrary value, (see below) at working loads, or to use the formula given below, from SABS 0100 , or that from BS 8007 (See also Appendix D and the crackwidth computer program supplied).

6.2 Crack Control by incremental moment.

Three approaches may be followed.
In the first two, the distribution of moments between the column band and the slab band should be taken as the standard one for flat slabs, i.e. for hogging moment 75% in the column band, and 55% for sagging moments. The incremental moment is the difference between the total moment, and the moment required to reduce the compression in the concrete adjacent to the reinforcement to zero.

In the first method, the incremental stress in the steel is controlled, and experience shows that crackwidth is satisfactory (See e.g. S.A. Bridge code). For a crackwidth of 0.1 mm, the Bridge Code states that the incremental stress should be limited to 75 MPa. For a crackwidth of 0.2 mm, the incremental stress should be 150 MPa. This is considered very conservative. However if there is appreciable prestress, it may not be possible to reach the recommended tensile stress and maintain compatibility of strains.

In the second method the incremental stress is calculated as the stress caused by the incremental moment. Then a computer program for crackwidth may be used, or the following formula.

\[
\text{Design surface crackwidth} = \frac{3a_{cr}E_m}{1 + 2\left[\frac{\sigma_{cr} - \sigma_{min}}{h - \delta}\right]}\]

(See Appendix D for an explanation of the formula)

In the third method the aim is to provide adequate reinforcing so that the cracks are well distributed. Rules are given in 6.2.2

6.2.1 Crackwidth

6.2.2 Bar spacing rules.

For un-prestressed slabs, the bar spacing rules given in SABS 0100 are adequate.

The Swiss approach is to limit crackwidth by an empirical formula specifying the minimum amount of ordinary reinforcement to distribute cracks.

For end spans

\[
P_s = 0.50 P_{pr}, \text{ but not less than 0.05%}
\]

where

\[
P_s = \text{Percentage of ordinary reinforcement (deformed HT steel)}
\]

\[
P_{pr} = \text{Percentage of prestressing steel}
\]

For internal spans, no minimum steel quantities are required. The steel required for the Ultimate limit state is assumed to be adequate.

Over columns, the minimum steel area of 0.3% must be provided over a width equal to the width of the column + three times the effective depth. Additional steel equal to 0.15% is required over the remainder of the column zone.

6.3 Deflection Control

Deflection may be controlled in the preliminary design by using the suggested span-depth rules, if at least one half of the dead + live load is balanced by prestress. Where the depth approaches the minimum, or conditions are such that excessive creep or temperature conditions may be expected, or loads are unusually heavy, the deflections should be calculated.

For final design the deflections should be calculated, making allowance for two-way action, creep and cracking.

6.3.1 Calculation of Deflection

The deflection calculated from the analysis may be used, using the gross moment of Inertia, and corrected using the ACI method (as proposed by Branson):

\[
I_{as} = \left(\frac{M_{ar}}{M_D}\right)^3 I_g + \left[1 - \left(\frac{M_{ar}}{M_D}\right)^3\right] I_{cr}
\]

where

\[
M_{ar} = \text{moment at first cracking of the concrete}
\]

\[
M_D = \text{design bending moment at midspan}
\]

\[
I_g = \text{gross moment of inertia}
\]

\[
I_{cr} = \text{cracked moment of inertia based on the transformed steel areas of both stressed and non-stressed steel. (bonded or unbonded)}
\]
For continuous members I_e can be modified as follows:

\[ I_e = 0.85(I_e)_{\text{centre span}} + 0.15(I_e)_{\text{support}} \]

for members continuous over one support.

and

\[ I_e = 0.70(I_e)_{\text{centre span}} + 0.15(I_e)_{\text{left}} + 0.15(I_e)_{\text{right}} \]

for members continuous both sides.

For cracked members the stiffness term (EI)_cr is relatively insensitive to the modular ratio.

The maximum deflection at the centre may be taken as the sum of the maximum deflections of two orthogonal frames.

This is conservative. A better method is to calculate the deflection in the column band in one direction, and the slab band in the other direction, and add them. However there are so many unknowns, (Elastic modulus, cracking, creep, shrinkage, etc.) that no method is entirely satisfactory. (See sample calculation in Appendix C)

Deflection may also be calculated using finite element programs, but unless the non-linear conditions are taken into account, the accuracy to be expected is not greater than for the frame type of analysis.

If a grillage program is used with correction made for cracking, the accuracy may be expected to be reasonable. The deflection due to creep should be taken into account. This is usually taken into account by using a reduced Young’s Modulus.

7.0 Ultimate Limit State

7.1 Redistribution of moments

Because a continuous beam or slab structure will only collapse in bending when 3 hinges form in a span, the attainment of ultimate moment at a section does not necessarily cause collapse. If the structure is able to form a plastic hinge, the load may be increased until full capacity is reached at other sections. For this reason it is permissible to re-distribute the moments for the ultimate limit state.

For prestressed concrete, SABS 0100 states that redistribution is permissible providing:

(a) Equilibrium between internal forces and external loads is maintained under each appropriate combination of design ultimate load.

(b) The reduction made to the maximum design moment (hogging or sagging), derived from an elastic maximum moments diagram covering all appropriate combinations of design ultimate load does not exceed 20%, except that in structures over 4 storeys high, where the frame provides lateral stability, the maximum reduction is 10%.

(c) Where the design moment is reduced, the neutral axis depth x should be checked to see that it is not greater than

\[ (\beta_d - 0.5)d \]

where

\[ \beta_d \]

is the ratio of moment at the section after re-distribution, to the maximum elastic moment.

This last rule will generally rule out re-distribution in members which do not have a low prestress level.

The ACI code allows re-distribution of hogging moments in prestressed members by up to 20%, depending on the concrete strength and reinforcement.

The committee recommends that 15% redistribution be permitted with no further calculation, but that if a larger redistribution is desired, the literature should be consulted.

7.2 Stresses

When tendons are bonded, they can reach their failure stress in bending in localized areas, at the ultimate limit state. Because unbonded tendons can stretch over their full length, the stress in the tendons at failure may not reach 'yield'. The incremental stress in the tendon, above that induced by the prestressing process, after allowing for losses, is due to the deflection of the slab increasing the effective length between anchorages. There are two suggested ways to calculate the ultimate limit state in bending.

These differ in that the Report 25 method ignores so-called 'secondary' moments, and the second suggested method, (which is recommended), takes them into account.
Prestressed Concrete Slabs  

7.2.1 Report 25 Method  
The method given in Report 25 is to calculate the factored dead load and (patterned) live loads moments, ignoring the effect of the prestress. The prestressing steel is considered as reinforcement (at a stress depending on relative area of the tendons), together with any non-prestressed reinforcement. The Ultimate MR of the section may be obtained from table 3.1 of Report 17, (which is the same as SABS 0100) and is reproduced below:

<table>
<thead>
<tr>
<th>Stress in tendons as a proportion</th>
<th>Ratio of depth of neutral axis to</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.025</td>
<td>1.23</td>
</tr>
<tr>
<td>0.05</td>
<td>1.21</td>
</tr>
<tr>
<td>0.10</td>
<td>1.18</td>
</tr>
<tr>
<td>0.15</td>
<td>1.14</td>
</tr>
<tr>
<td>0.20</td>
<td>1.11</td>
</tr>
</tbody>
</table>

\[ f_{pb} \] may not exceed 70% of \( f_{pu} \)

Where MR = \( A_{ps} f_{p} b( d-x/2) \)  
The stress \( f_{p} \) and the depth of neutral axis \( x \) are obtained from the table. If bonded steel is provided, as is essential over columns, the ratio in the first column of the table is (\( f_{pc} A_{ps} + 0.87 f_{s} A_{s} \))/(\( f_{pu} b d \)).

The increase may be taken from the above table, or from Concrete Society Report 23 on partial prestressing, which gives:

\[ f_{pb} \text{ not greater than } f_{pc}/1.15 \]

\[ f_{pb} = f_{pc} + \Delta f_{u} \]

where \( f_{pc} \) is the stress after losses, and

\[ \Delta f_{u} = \frac{d}{17L} \times E_{p} \] at midspan, and

\[ \Delta f_{u} = 2 \times \frac{d}{17L} \times E_{p} \] at supports.

\( d \) is the effective depth  
\( L \) is the tendon length between anchorages  
\( E_{p} \) is the Young's modulus of the prestressing steel.

It should be pointed out that for an unbonded tendon, the stress in the tendon is not related to the strain in the adjacent concrete, and normal bending theory does not really apply.

In addition, it does not seem logical that the tendons over supports would have a greater stress than at mid-span.

7.2.2 Factored upward prestress load method  
This method regards the tendons as external forces on the slab, and is therefore compatible with the design assumptions of the balanced load method.

The method is to apply the factored dead load and (patterned) live loads and apply lateral loads due to an increased prestress in the tendons (due to deflection) in the same way as in 2.2. The additional prestress may be taken from 7.2.1. If the second method is used, as it is not logical for an unbonded tendon to have radically different forces at different points along its length, it would be logical to assume an overall increase of \( 1.5dE_{p}/17L \) for calculating the lateral loads due to prestress.

The overall prestress compressive force is applied to the section, and the decompression moment calculated. This is the moment required to cause zero stress in the concrete at the level of the prestressing steel. Additional reinforcement, at a stress of \( f/1.15 \), is supplied to meet the ultimate load conditions.

At the Ultimate limit state the stress in the concrete, using either a rectangular stress block or the parabolic stress block
of SABS 0100, should not exceed 0.45 f\textsubscript{cu} which includes the material factor \( Y_{m} \).

At the supports, not less than 0.15\% of the gross cross-sectional area based on the width of the column plus twice the slab depth each side of the column must be provided.

(See also the requirement in 2.0 for exterior spans to provide enough steel for the condition that 50\% of the prestress is lost.)

At external columns the non-prestressed reinforcement must be bent down and well bonded.

8.0 Shear

Shear need only be calculated for the ultimate limit state. Most codes, now and in the past, calculated the shear stress at \( h/2 \) or \( h \) from the column face. The ACI code uses \( h/2 \), but the allowable shear stress for punching is twice that for one way shear. (The ACI code allows a maximum shear stress of \( 3 \sqrt{(f_{c})} \) for one way slabs, and \( 6 \sqrt{(f_{c})} \) for two-way slabs, in psi, equivalent to \( 0.25 - 0.5 \sqrt{(F_{c})} \) in Mpa.)

The failure surface where a punching shear failure occurs in a section unreinforced for shear, is a pyramid or cone with an angle of 25\° to 30\° to the horizontal. For this reason CP110 some years ago changed the method of calculating punching shear stress to a perimeter of 1.5\( h \) from the face of the column or capital. This had the result that the calculated shear stress in punching shear is approximately the same as for one way shear. The perimeter was changed in the recent amendment from a rounded rectangle, to a plain rectangle.

However the unmodified SABS0100 and CP110 method does not give good results for thin slabs, which are stronger than the method would predict, or for thick slabs, which are weaker, and a correction factor has been incorporated. (The SABS and BS codes also take account of the percentage of reinforcement, which is neglected in the ACI code.)

In SABS 0100 the shear resistance is given as:

\[
v_{c} = 0.171 \left( f_{cu} \times 100 \times A_{se}/bd \right)^{1/3} \times (400/d)^{1/4}
\]

(Units MPa and mm)

Where \( A_{se} \) is the equivalent area of reinforcement passing through the critical perimeter, and is equal to \( A_{n} + A_{ps} \times f_{psu}/410 \) MPa.

where \( A_{n} \) is the area of non-prestressed high tensile reinforcement, and \( A_{ps} \) is the area of the prestressing steel, and \( f_{psu} \) is the characteristic strength of the prestressing tendon.

**It should be noted that in the South African Code the correction factor applies for all depths**, whereas BS8110 has a cut-off point for slabs deeper than 400mm. This will affect deep slabs, such as foundation rafts. \( f_{cu} \) is the characteristic strength, \( d \) is the effective depth and \( b \) is the width of the critical perimeter.

\( A_{se}/bd \) must not be taken greater than 0.03 (ie. 3\%)

The allowable stress may be different on adjacent sides and the average value is calculated.

This formula applies to a DL factor of 1.2, as used in the new loading code. The old allowable stress has accordingly been reduced by 10\%.

The effective load for which the shear is to be calculated must be derived from the analysis, and this must be multiplied
by a factor to allow for the effect of moment transfer to the columns.

At internal columns the effective shear (due to the effect of transfer of moment to the columns)

\[ V_{\text{eff}} = V (1+1.5M/Vx) \]

but where spans are approximately equal, 
\[ V_{\text{eff}} = 1.15V \] may be assumed. 

Where \( V \) is the total shear from the elastic analysis for a particular load case, \( M \) is the moment transferred to the column for the same load case. This may be reduced by 30% if the equivalent frame analysis is used, and if there is pattern loading, but not otherwise. \( x \) is the length of the side of the shear perimeter parallel to the axis of bending.

At external columns where bending is about an axis parallel to the free edge 

\[ V_{\text{eff}} = 1.25V \]

For external columns where bending is about an axis perpendicular to the free edge 

\[ V_{\text{eff}} = V (1.25 + 1.5M_t/Vx) \]

or 1.4 \( V \) may be used for approximately equal spans.

\[ M_t = \text{transverse } M, \text{ which may be reduced by 30\% if a frame analysis and pattern loading has been used. (In the old code, the slab moments could only be adjusted by 15\%)} \]

The effective shear \( V_{\text{eff}} \) at any perimeter away from the face of the column may be reduced by the vertical component of the prestressed tendons lying within a width 0.7 \( d \) each side of the column face. (It should be noted that for cracked prestressed beams, the vertical component of the tendon force may not be assumed to reduce the effective shear.) Where it is desired to check the shear stress at a perimeter closer than 1.5\( h \), the allowable shear stress may be increased by a factor 1.5\( h/a_0 \) where \( a_0 \) is the distance of the perimeter from the column face. The shear perimeter is reduced if there are holes near the column. (See Diag. 8.2)

In all cases the effective shear should be checked in both directions, and the average taken.

If the shear stress at the control perimeter exceeds the permissible, shear reinforcement must be provided.

8.1 Provision of shear reinforcement in a failure zone (for definition of zone see Diag. 8.3). If \( v \) exceeds \( v_c \), shear reinforcement may be provided in slabs exceeding 150 mm thick to increase the shear resistance in accordance with the formula:-

\[ \Sigma A_{sv} \sin \alpha \geq \frac{(v-v_c)ud}{0.87f_s} \]

\( A_{sv} \) is the area of shear reinforcement, 
\( \alpha \) is the angle between the shear reinforcement and the plane of the slab, (usually 90\°) 
\( d \) is the effective depth. and \( u \) the shear perimeter. 
\( v-v_c \) must not be taken as less than 0.4 MPa. 
For slabs greater than 200mm thick, \( f_s = f_y \) (the characteristic strength of the shear reinforcement), or 425 MPa, whichever is less.
For slabs between 150 mm and 200 mm thick, $f_s = f_y v (h-150)/50$ or $425*(h-150)/50$, whichever is less.

The reason for reducing the allowable steel stress in shear for depths less than 200 mm thick, is that for thinner slabs shear reinforcement is not as effective. For slabs less than 150 mm thick, shear reinforcement is not considered effective.

**Diagram 8.3**

(There is a system popular in the USA which uses studs welded to a metal strip. This seems to be effective in shallower slabs). The shear reinforcement should be distributed evenly around the zone on at least 2 perimeters. The spacing around the perimeter should not exceed $d$. In assessing the shear reinforcement required, shear reinforcement within the zone provided to reinforce other zones may be taken into account.)
8.2 Waffle slabs.  
Ribs are designed as beams, and the solid section as for ordinary flat slabs. (see 5.4.3)

8.3 Openings  
Where openings are near a column, the shear strength is adversely affected. See Diag. 8.2 for recommendations on how to deal with holes near columns.

9.0 Detailing  
9.1 Tendons  
9.1.1 Layout  
As pointed out in 3.4, there are a number of different possible tendon layouts. (see Diags. 3.1 and 3.2). The principle to be followed is that tendons exert a load on the slab, upwards where they are concave upwards, and downwards where the tendons are concave downwards. The upward loads due to tendons should, as far as possible, balance the downward loads due to the self weight, live load, and the downward load from other tendons. The downward load from tendons should be taken by supports, or by other tendons. The transfer of load is illustrated diagrammatically in Diag. 9.1  
Where there is an irregular column layout there is an advantage in being able to trace the way the load is carried to the supports, but even for regular column layouts the tendon layout in Diag. 3.2 simplifies the construction sequence, and enables the maximum drape to be obtained for most tendons.

9.1.2 Notation  
The notation developed in the U.K., and shown in Diag. 9.2 should be used, and Diagram 9.2 should be included on the tendon layout drawing.

9.1.3 Tendon spacings  
9.1.3.1 Maximum spacing  
There is no requirement for maximum tendon spacing, in that tendons may be arranged in bands, but where tendons are spaced out, it is considered good practice to space them at not more than 6 slab depths apart, or 8 slab depths if adequate non-prestressed reinforcement is provided to control cracking. A minimum of two tendons in each direction should pass through the critical shear area at the column. For punching shear it is advantageous to position more than 50% of the tendons through the critical shear area.

9.1.3.2 Minimum spacing  
Where tendons are grouped in bands, the clear spacing should be large enough to ensure that proper compaction of the concrete can be effected, and to allow room for non-prestressed reinforcement, so that the tendon spacing should bear some relation to the reinforcement spacing. An absolute minimum clear space of 75 mm for a vibrator should be provided between each group of 3 tendons, and the tendons between the 75mm openings should be spaced at not less than 1.5 times the aggregate size, i.e. 30 mm if 19 mm aggregate is used.  
At anchorages the grouped tendons are splayed out in plan, to enable the anchorages to be placed. Where tendons are deviated in plan, it must be realized that forces are applied to the structure, and the designer must satisfy himself that these forces are properly catered for. The same applies where tendons are deviated round openings.

9.1.4 Profiles and stools  
Various profiles are used in practice, including parabolic and 'harped' or straight line profiles.
Where straight line profiles are used, deviated at the quarter points of the span, the tendons are tied to the bottom reinforcement mat, and stools need only be provided over the remaining 50% of their length.

Stools should be provided at a minimum spacing of one meter, to achieve a smooth accurate profile. The most commonly used stool is fabricated from welded steel mesh which is cut and bent to give the required height. Stools are available in heights from 70 mm, increasing in increments of 10 mm. Not less than 6 mm wire should be used for stools from 70 mm to 150 mm high.

For stools 160 mm to 250 mm high 8 mm wire should be used, and for stools 260 mm to 350 mm high 10 mm wire should be used.

Stools are available in lengths of 1200 mm and 2400 mm. It is recommended that a continuous length of stool be provided under banded tendons, while individual stools (min 200 mm long) are used for single tendons. To ensure stability of the stools for individual tendons during concreting, Y10 lacer bars should be tied to the row of stools at the span quarter points for the whole length of the row of stools.

9.2 Reinforcement

9.2.1 Minimum non-prestressed reinforcement

A minimum area of non-prestressed reinforcement should be provided in the top at all supports, in the bottom of all column bands, and in the bottom of external slab bands. (see also crack control - 6.1, and the requirements for the Ultimate Limit State in external spans in 2.0)

9.2.1.1 Positive (sagging) moment areas.

The minimum area should be 0.075% of the gross cross-sectional area of the band. (But see also 2.0). Fifty percent of the bars should have a minimum lap of 300 mm at support lines. Where the concrete is calculated to be in tension, enough reinforcement should be provided to limit the incremental stress, or to control the crack width by other methods. The bars which do not extend to the support lines should have a minimum length of half the span. Lapping the bars at supports gives a post-failure strength due to catenary action in the case of a catastrophic failure.

9.2.1.2 Negative moment areas.

The minimum area of non-prestressed reinforcement provided over supports is 0.15% wh, where w is the column width plus 4 times h, and h is the overall slab depth. This reinforcement is to be spread over a width equal to the column width plus 1.5h on either side of the column. Not less than 4 Y12 bars at a maximum spacing of 200 mm should be provided. The reinforcement should extend at least one sixth of the clear span on each side of the support.
9.2.2 Shrinkage and temperature reinforcement

Where prestressing is in one direction only, the normal percentages of reinforcement for non-prestressed slabs must be provided in the other direction. These are:

- 0.12% if High Tensile steel is used
- 0.27% if mild steel is used.

Maximum spacing should not exceed 5 times the slab thickness, or 500 mm, whichever is smaller.

9.2.3 Reinforcement around openings

Horizontal deviations of tendons which may be necessary to avoid openings, chases, inserts, etc., should have a radius of curvature not less than about 6.5 m to avoid excessive lateral forces which might cause cracking. They should preferably not be deviated through an angle more than that corresponding to a slope of 6:1. At least 300 mm of straight tendon should extend beyond the corners of the opening, and at least 75 mm clearance should be provided between the tendon and the edge of the opening. In cases where the deviation exceeds a slope of 6:1, Y10 hairpin bars, at not more than 150 mm centres, should be used to transmit the lateral forces from the tendons to the surrounding concrete.

For larger openings, where it is necessary to terminate some tendons, the top and bottom of the slab adjacent to the opening should be reinforced with diagonal bars at the corners, and around the periphery of the opening. Openings should preferably be located in the slab band areas of two-way slabs to reduce the effect of the opening on the shear capacity of the slab.

9.2.4 Bursting reinforcement

The analysis of end blocks may be made by using SABS 0100 Clause 4.8.5.

Hairpin bars should be provided perpendicular to the edge of the slab and bursting reinforcement parallel to the edge of the slab. They should be well bonded at corners, if the tendons are at all close to the corner. R8 spirals are commonly used around the tendons, just behind the anchorage, and are considered part of the bursting reinforcement.
From the basic equation for parabolas \( y = kx^2 \), consider the parabolas AB, BCD, and DE:

\[ c_1 = k_1 a_1^2 \quad \text{and} \quad c_2 = k_2 a_2^2 \]  

(1)

Letting \( B_1 = b_1 - b \) \( a_1 \) and \( B_2 = b_3 - b_2 \),

Then \( (B_1 - c_1) = k (X-a_1)^2 \) \( (B_2 - c_2) = k (L-X-a_2)^2 \)  

(2)

The slope \( \frac{dy}{dx} \) and the parabolas are tangential at B and D.

\[ \phi_1 = 2k_1 a_1 \quad \text{and} \quad \phi_2 = -2k_2 a_2 \]

\[ \phi_1 = -2k (X-a_1) \quad \text{and} \quad \phi_2 = 2k (L-X-a_2) \]  

but

\[ k_1 = -\frac{k(X-a_1)}{a_1} \quad \text{and} \quad k_2 = -\frac{k(L-X-a_2)}{a_2} \]  

(3)

From Equations (1) and (3):

\[ c_1 = -k(X-a_1) \quad \text{and} \quad c_2 = -k(L-X-a_2) \]

From Equation (2)

\[ B_1 + k(X-a_1)a_1 = k(X-a_1)^2 \]

\[ B_2 + k(L-X-a_2)(L-a_2) = (L-X-a_2)^2 \]

\[ \frac{1}{k} = \frac{(K-a_1)^2 - (X-a_1)}{B_1} \quad \text{and} \quad \frac{1}{k} = \frac{(L-X-a_2)^2 - (L1-X-a_2)}{B_2} \]

These combine to give the quadratic:

\[ 1X^2 + mX + n = 0 \]

where \[ l = (b_3 - b_1) \]

\[ m = (2L - a_2)(b_1 - b_2) - a_1(b_3 - b_2) \]

\[ n = -(b_1 - b_2)(L-a_2)L \]

with the solution \[ X = \frac{-m + \sqrt{m^2 - 4ln}}{2l} \]  

If \( l = 0 \) then \( X = -n/m \)

Then \( c_1 = (b_1 - b_2)a_1/X \) and \( c_2 = (b_3 - b_2)a_2/(L - X) \)

The drape is given by: \[ \text{Drape} = \frac{(b_2 - b_1) (L-a_1-a_2)^2}{4X (X-a_1)} \]
For a beam with span L, and uniform load w, stiffness EI, the deflection is

\[ \delta = \frac{k w L^4}{EI} \]

where \( k = \frac{5}{384} \) for simply supported beams and \( k = \frac{1}{384} \) for fully fixed ends.

Approximately \( \frac{1.4}{384} \) for internal spans
\( k = \frac{2.6}{384} \) for external spans of flat slabs.

The effective load causing deflection may be taken as:

\[ w_{eff} = (\text{Creep factor}) \times \text{Permanent Load} + \text{Live Load} \]

and if it is assumed that \( \frac{\delta}{L} = \text{Constant} = \frac{k w L^4}{EI} \)
(for spans less than 10 m),

\[ I = h^3 \]

Therefore

\[ \text{Constant} = \frac{k w_{eff} L^3}{Eb^3} \]

and

\[ \frac{L}{h} \ll \left[ \frac{E}{w_{eff}} \right]^{1/3} \]

If the proportion of permanent load balanced by prestress is Q, the effective load:

\[ w_{eff} = (\text{creep factor} \times (1-Q)PL + LL) \]

and if the creep factor is taken to be 35 and to allow for the fact that the prestress might not be fully effective, the proportion of permanent load balanced is multiplied by 0.95, then:

\[ \frac{L}{h} = \frac{kE^{1/3}}{(PL(3.5-3.32Q)+LL)^{1/3}} \]

To allow for the effect of end spans, different E's than the one assumed (for class 35 concrete) and to allow for the possibility that slabs may be in a situation which is unfavourable for deflection due to a greater likelihood of cracking (eg exposed to the sun), factors \( K_1 \) to \( K_4 \) are introduced.

\[ K_1 = \text{Factor for end span or internal span} \quad K_1 = 0.9 \]
For internal span

\( K_2 = 1.0 \) if cracking is unlikely and \( K_2 = 0.95 \) if it is.

\( K_3 = \sqrt[3]{\frac{E}{E_{35}}} \)

\( K_4 = 1.0 \) for flat plates

\( K_4 = 1.15 \) if adequate drops are used

To allow for the fact that \( L/h \) cannot be infinite even if the load is all balanced, and also to allow for the fact that the shear affects the depth, set

\[
\frac{L}{h} = \left( C_1 + \frac{C_2}{[P(L \times 3.5 + LL) + LL]}^{\frac{1}{3}} \right) \cdot K_1 \cdot K_2 \cdot K_3 \cdot K_4
\]

and after examining a number of different slabs, and calculating the constants, one gets:

\[
\frac{L}{h} = \left( 14 + \frac{53}{(P(L \times 3.5 + LL - Q(L \times 3.5))^{\frac{1}{3}}} \right) \cdot K_1 \cdot K_2 \cdot K_3 \cdot K_4
\]

The nomogram given in Report 25 (Diag A) which presumably assumes that the prestress used conforms with the recommendations for balancing \( \frac{1}{2}(DL + LL) \), shows an example for \( L=8m, Fc = 35 \text{ mPa}, LL=3.5 \text{ kPa} \) which gives a depth of 175mm and \( L/h = 45.7 \). This is actually outside its own recommendations of \( L/h = 34 \) to 42 for normal loading.

Substituting the above values in the above formula, and assuming that half of \( (DL + LL) \) is balanced, and that the slab thickness is 180mm as an initial guess

Say permanent load = 0.18 x 24 = 4.32, with no permanent live load

Then \( Q = 4.32 \times 2/(3.5+4.32) = 0.905 \) (proportion of DL balanced)

from the formula \( L/h = (14+53/1.92) \times 1.01 \times 44.32 \) and 8.0/44.32 = 0.18 so depth = 180 mm.